

Assam Academy of Mathematics
Assam Mathematics Olympiad 2026
Category II (Classes VII - VIII)
30th August 2026

Full marks : 100

Time : 3 hours

There are 18 questions. Questions 1 to 5 carry 2 marks each. Questions 6 to 13 carry 5 marks each. Questions 14 to 18 carry 10 marks each. Calculators or any other electric or digital devices are strictly prohibited.

ইয়াত 18 টা প্ৰশ্ন আছে। 1 ৰ পৰা 5 লৈ প্ৰতিটো প্ৰশ্নত 2 নম্বৰকৈ আছে। 6 ৰ পৰা 13 লৈ প্ৰতিটো প্ৰশ্নত 5 নম্বৰকৈ আছে। আৰু 14 ৰ পৰা 18 লৈ প্ৰতিটো প্ৰশ্নত 10 নম্বৰকৈ আছে। কেলকুলেটৰ বা অন্য কোনো বৈদ্যুতিক বা ডিজিটেল সঁজুলি সম্পূৰ্ণৰূপে নিষিদ্ধ।

There may be various other ways of solutions than those shown here. Queries or suggestions regarding the solutions can be mailed to mail@aamonline.in
ইয়াত দেখুওৱা ধৰণবিলাকৰ বাহিৰেও প্ৰশ্নবোৰৰ সমাধানৰ আন বিভিন্ন উপায় থাকিব পাৰে। সমাধানবোৰৰ বিষয়ে কিবা প্ৰশ্ন বা পৰামৰ্শ থাকিলে mail@aamonline.in লৈ মেইল কৰিব পাৰে।

1. Find the highest common factor (HCF) and least common multiple (LCM) of 2026 and 20262026.

2026 আৰু 20262026 ৰ গৰিষ্ঠ সাধাৰণ উৎপাদক (গসাউ) আৰু লঘিষ্ঠ সাধাৰণ গুণিতক (লসাগু) নিৰ্ণয় কৰা।

Ans : Observe that $20262026 = 20260000 + 2026 = 2026 \times (10000 + 1) = 2026 \times 10001$. So, to find the HCF and LCM of 2026 and 20262026, it is not necessary to factorize 2026 any further. Required HCF is 2026 and the required LCM is $2026 \times 10001 = 20262026$.

2. A natural number is called *good-looking* if each of its digits is odd. For example, 1357 is good-looking, whereas 1235 is not. How many four digit good-looking numbers are there?

এটা স্বাভাৱিক সংখ্যাক সুশোভন সংখ্যা বুলি কোৱা হয় যদিহে ইয়াৰ আটাইকেইটা অংক অযুগ্ম। উদাৰণস্বৰূপে, 1357 সংখ্যাটো এটা সুশোভন সংখ্যা, অথচ 1235 সংখ্যাটো নহয়। চাৰিটা অংকবিশিষ্ট মুঠ কিমানটা সুশোভন সংখ্যা থাকিব নিৰ্ণয় কৰা।

Ans : The possible odd digits are 1, 3, 5, 7, and 9, so there are 5 choices for each digit. Since every choice for the thousands digit is non-zero, each of the four positions can be filled independently in 5 ways. Therefore, the number of four-digit good looking numbers is $5 \times 5 \times 5 \times 5 = 5^4 = 625$.

3. A book has 1013 leaves, and its pages are numbered consecutively from 1 to 2026. Victor tears out 17 leaves and adds all the page numbers written on them. Can the sum be 2026? Give reasons for your answer.

এখন কিতাপত 1013 টা পাত আছে আৰু ইয়াৰ পৃষ্ঠাসমূহ ক্ৰমাগতভাৱে 1 ৰ পৰা 2026 লৈ সংখ্যাৰে চিহ্নিত কৰা আছে। ভিক্টৰে কিতাপখনৰ 17 টা পাত ফালি পেলালে আৰু তাত লিখা থকা পৃষ্ঠা নম্বৰসমূহ যোগ কৰিলে। এই যোগফলটো 2026 হ'ব পাৰেনে? তোমাৰ উত্তৰটোৰ সপক্ষে যুক্তি দৰ্শোৱা।

Ans : No. Each leaf contains two consecutively numbered pages, one odd and the other even, so the sum of the two page numbers on every leaf is odd. Therefore, the sum obtained from 17 leaves is the sum of 17 odd numbers, which must be odd. Since 2026 is even, Victor could not have obtained 2026 as the sum.

4. At a book fair, a stall has 5 different storybooks, 3 different puzzle books, and 4 different colouring books. In how many ways can a student buy two books of different types ?

এখন গ্ৰন্থ মেলাৰ এখন বিপনীত 5 খন ভিন ভিন গল্পৰ কিতাপ, 3 খন ভিন ভিন সাঁথৰৰ কিতাপ আৰু 4 খন ভিন ভিন ৰঙ সম্বন্ধীয় কিতাপ আছে। এগৰাকী শিক্ষাৰ্থীয়ে কিমান ধৰণে দুখন বেলেগ বেলেগ ধৰণৰ কিতাপ কিনিব পাৰিব?

Ans : There are three possible cases. A student may buy one storybook and one puzzle book in $5 \times 3 = 15$ ways, one storybook and one colouring book in $5 \times 4 = 20$ ways, or one puzzle book and one colouring book in $3 \times 4 = 12$ ways. Since these cases do not overlap, the required number of ways is $15 + 20 + 12 = 47$.

5. Amit spends $\frac{5}{14}$ of his monthly income on food, $\frac{1}{10}$ on monthly bills, $\frac{8}{21}$ on other expenses and saves the rest. Preeti spends $\frac{4}{15}$ of her income on food, $\frac{1}{6}$ on monthly bills, $\frac{3}{7}$ on other expenses and saves the rest. Who saves a larger portion of their income ?

অমিতে নিজৰ মাহিলী আয়ৰ $\frac{5}{14}$ অংশ খাদ্যৰ বাবে, $\frac{1}{10}$ অংশ মাহিলী বিলসমূহৰ বাবে, $\frac{8}{21}$ অংশ অন্যান্য খৰচৰ বাবে ব্যয় কৰে আৰু অৱশিষ্ট অংশখিনি জমা থয়। প্ৰীতিয়ে নিজৰ মাহিলী আয়ৰ $\frac{4}{15}$ অংশ খাদ্যৰ বাবে, $\frac{1}{6}$ অংশ মাহিলী বিলসমূহৰ বাবে, $\frac{3}{7}$ অংশ অন্যান্য খৰচৰ বাবে ব্যয় কৰে আৰু অৱশিষ্ট অংশখিনি জমা থয়। এওঁলোকৰ ভিতৰত কোনে নিজৰ মাহিলী আয়ৰ অধিক অংশ জমা থয়?

Ans : Amit spends $\frac{5}{14} + \frac{1}{10} + \frac{8}{21} = \frac{5 \times 15 + 1 \times 21 + 8 \times 10}{210} = \frac{176}{210}$ part. So, he saves $1 - \frac{176}{210} = \frac{34}{210}$ part.

Preeti spends $\frac{4}{15} + \frac{1}{6} + \frac{3}{7} = \frac{4 \times 14 + 1 \times 35 + 3 \times 30}{210} = \frac{181}{210}$ part. So, she saves $1 - \frac{181}{210} = \frac{29}{210}$ part.

Since $\frac{34}{210} > \frac{29}{210}$, so Amit saves more portion of his income than Preeti.

6. The sum of 9 consecutive even numbers is 5040. Find the smallest and the largest of these numbers.

9 টা ক্ৰমিক যুগ্ম সংখ্যাৰ যোগফল 5040। এই সংখ্যাসমূহৰ আটাইতকৈ সৰু আৰু আটাইতকৈ ডাঙৰ সংখ্যা দুটা নিৰ্ণয় কৰা।

Ans : Since there are nine consecutive numbers, we consider the middle number as x . Then the numbers are $x - 8, x - 6, x - 4, x - 2, x, x + 2, x + 4, x + 6, x + 8$. By question,

$$(x - 8) + (x - 6) + (x - 4) + (x - 2) + x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 5040$$

so that $9x = 5040$ i.e. $x = 560$. Hence, the smallest number is $560 - 8 = 552$ and the largest is $560 + 8 = 568$.

Alternatively, one could assume the numbers as $x, x + 2, x + 4, x + 6, x + 8, x + 10, x + 12, x + 14, x + 16$ so that $9x + 72 = 5040$ i.e. $x = \frac{4968}{9} = 552$.

7. Can you find distinct real numbers a, b, c such that $a^2(b - c) + b^2(c - a) + c^2(a - b) = 0$? Justify your answer.

তুমি এনেকুৱা তিনিটা পৃথক বাস্তৱ সংখ্যা a, b, c বিচাৰি উলিয়াব পাৰিবানে যাতে $a^2(b - c) + b^2(c - a) + c^2(a - b) = 0$? তোমাৰ উত্তৰটোৰ যুক্তিযুক্ততা বিচাৰ কৰা।

Ans : We have,

$$\begin{aligned} & a^2(b - c) + b^2(c - a) + c^2(a - b) = 0 \\ \Rightarrow & a^2(b - c) + b^2c - bc^2 + c^2a - b^2a = 0 \\ \Rightarrow & a^2(b - c) + bc(b - c) + a(c^2 - b^2) = 0 \\ \Rightarrow & a^2(b - c) + bc(b - c) - a(b - c)(b + c) = 0 \\ \Rightarrow & (b - c)\{a^2 + bc - a(b + c)\} = 0 \\ \Rightarrow & (b - c)(a^2 - ab + bc - ac) = 0 \\ \Rightarrow & (b - c)\{a(a - b) - c(a - b)\} = 0 \\ \Rightarrow & (b - c)(a - b)(a - c) = 0 \end{aligned}$$

But the product on LHS can be 0 if and only if $a = b$ or $b = c$ or $c = a$. So, for distinct a, b, c the expression cannot be 0.

8. The first 25 odd numbers are written on a board. One of these numbers is erased. The sum of all the numbers remaining on the board is a perfect square. Which number was erased?

এখন বোর্ডত প্ৰথম 25 টা অযুগ্ম সংখ্যা লিখি থোৱা আছে। ইয়াৰে এটা সংখ্যা মচি পেলোৱা হল। বোর্ডখনত থকা অৱশিষ্ট সংখ্যাকেইটাৰ যোগফল হৈছে এটা পূৰ্ণ বৰ্গ সংখ্যা। বোর্ডখনৰ পৰা কোনটো সংখ্যা মচি পেলোৱা হৈছিল?

Ans : The sum of the first 25 odd numbers i.e. $1, 3, 5, \dots, 49$ is $25^2 = 625$. Since the erased number is at least 1 and at most 49, the remaining sum lies between $625 - 49 = 576$ and $625 - 1 = 624$. The only perfect square in this range is $576 = 24^2$. Therefore, the erased number is $625 - 576 = 49$.

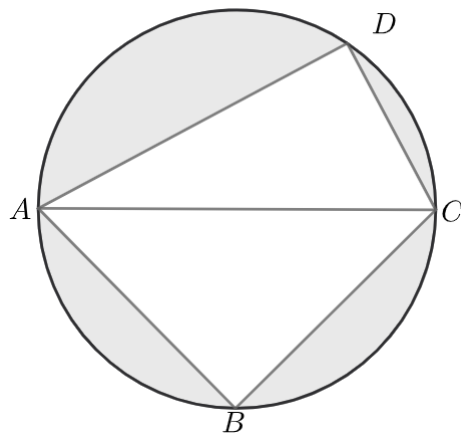
9. Which number is greater : 31^{11} or 17^{14} ? Justify your answer without actually calculating the values.

31^{11} আৰু 17^{14} এই সংখ্যা দুটাৰ কোনটো ডাঙৰ? সংখ্যা দুটাৰ প্ৰকৃত মান নিৰ্ণয় নকৰাকৈ তোমাৰ উত্তৰটোৰ যুক্তিযুক্ততা বিচাৰ কৰা।

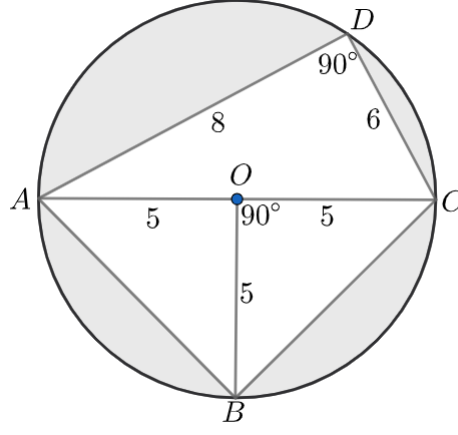
Ans : Since $31 < 32$, we have $31^{11} < 32^{11}$. Also, $32 = 2^5$, so $32^{11} = (2^5)^{11} = 2^{55}$. Now $2^{55} < 2^{56}$, and $2^{56} = (2^4)^{14} = 16^{14}$. Since $16 < 17$, we have $16^{14} < 17^{14}$. Therefore, $31^{11} < 32^{11} = 2^{55} < 2^{56} = 16^{14} < 17^{14}$. Hence, 17^{14} is greater.

10. In the following figure, $ABCD$ is a quadrilateral inscribed in a circle of radius 5 cm and AC is a diameter of the circle. Further, $AB = BC$ and $CD = 6$ cm. Find the area of the shaded region.

তলৰ চিত্ৰটোত দেখুওৱাধৰণে $ABCD$ চতুৰ্ভুজটো AC ব্যাসযুক্ত আৰু 5 চেমি ব্যাসাৰ্ধবিশিষ্ট বৃত্ত এটাৰ ভিতৰত আছে। তাৰোপৰি, $AB = BC$ আৰু $CD = 6$ চেমি। তেখেত চিত্ৰটোৰ ৰঙীণ ঠাইখিনিৰ কালি নিৰ্ণয় কৰা।



Ans : Let O be the centre of the circle. Since $AB = BC$, so BO is the altitude of $\triangle ABC$. Since the radius of the circle is 5 cm, so $BO = 5$ cm. Also, AC is the diameter. So, $AC = 2 \times 5 = 10$ cm. Hence, area of $\triangle ABC = \frac{1}{2} \times AC \times BO = \frac{1}{2} \times 10 \times 5 = 25$.

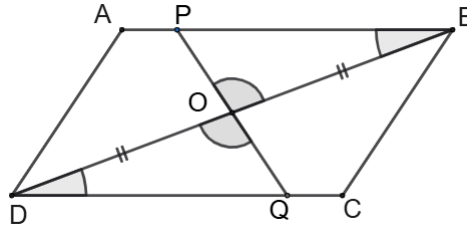


Now, $\triangle ADC$ is right angled at D . So, by Pythagoras theorem $AD^2 + CD^2 = AC^2$ i.e. $AD^2 + 6^2 = 10^2$ i.e. $AD^2 = 64$ i.e. $AD = 8$. Thus, the area of $\triangle ADC$ is $\frac{1}{2} \times AD \times DC = \frac{1}{2} \times 8 \times 6 = 24$. Also, the area of the circle is $\pi \times 5^2 = 25\pi$. Thus, required area of the shaded region is $25\pi - 25 - 24 = 25\pi - 49$.

11. Let $ABCD$ be a parallelogram and O be the mid-point of the diagonal BD . P and Q are points on AB and CD respectively such that P, O, Q are collinear. Show that $PO = OQ$.

ধৰা হ'ল, $ABCD$ এটা সামান্তৰিক আৰু O হৈছে BD কৰ্ণৰ মধ্যবিন্দু। P আৰু Q হৈছে AB আৰু CD ৰেখাখণ্ডৰ ওপৰত থকা এনেকুৱা দুটা বিন্দু যাতে P, O, Q বিন্দু তিনিটা একে ৰেখীয় হৈ পৰে। দেখুওৱা যে $PO = OQ$ ।

Ans : Since P, O, Q are collinear, so POQ is a straight line segment.



In $\triangle BOP$ and $\triangle DOQ$, we have

$$\angle PBO = \angle QDO \quad (\text{Alternate angles as } AB \parallel DC)$$

$$BO = OD \quad (\text{Since } O \text{ is the mid-point of } BD)$$

$$\angle BOP = \angle DOQ \quad (\text{Vertically opposite angles})$$

Thus, $\triangle BOP \cong \triangle DOQ$. Hence, $PO = OQ$.

12. Find the unit digit of the number $1^2 + 2^2 + 3^2 + \dots + 99^2$.

$1^2 + 2^2 + 3^2 + \dots + 99^2$ এই সংখ্যাটোৰ এককৰ স্থানত থকা অংকটো নিৰ্ণয় কৰা।

Ans : Adding 0^2 does not change the sum, so we may consider $0^2 + 1^2 + 2^2 + \dots + 99^2$. Divide these numbers into the ten groups 0 to 9, 10 to 19, ..., and 90 to 99 i.e.

$$(0^2 + 1^2 + 2^2 + \dots + 9^2) + (10^2 + 11^2 + \dots + 19^2) + \dots + (90^2 + 91^2 + \dots + 99^2).$$

Note that we just need the unit digit. So we don't have to compute the actual sum. Instead, we just compute the unit digits, add them and then extract only the unit digit from the sum and continue this way. Observe that the unit digits of $1^2, 11^2, 21^2, \dots$ are the same i.e. 1. Similarly, the unit digits of $2^2, 12^2, 22^2, \dots$ are the same i.e. 4. The unit digits of

$0^2, 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2$ are 0, 1, 4, 9, 16, 25, 36, 49, 64, 81. Adding, we get $0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 205$, so that the unit digit is 5. Thus in every group, the sum of squares will have the same unit digit 5. Thus, the ten groups together have the same unit digit as $10 \times 5 = 50$. Therefore, the last digit of $1^2 + 2^2 + \dots + 99^2$ is 0.

Those familiar with modular arithmetic, can observe that

$$(0^2 + 1^2 + 2^2 + \dots + 9^2) \equiv 0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 \pmod{10} \equiv 5 \pmod{10} \text{ so that}$$

$$(0^2 + 1^2 + \dots + 9^2) + (10^2 + 11^2 + \dots + 19^2) + \dots + (90^2 + 91^2 + \dots + 99^2) \equiv 5 \times 10 \pmod{10} \equiv 0 \pmod{10}.$$

13. What day of the week will be on 5th September 2050 ? Justify your answer.

2050 চনৰ ৫ ছেপ্টেম্বৰ তাৰিখে সপ্তাহটোৰ কি বাৰ হ'ব? তোমাৰ উত্তৰটোৰ যুক্তিযুক্ততা বিচাৰ কৰা।

Ans : We know 5th September 2026 is Saturday. There are $2050 - 2026 = 24$ complete years till 5th September 2050. Further, 2028, 2032, 2036, 2040, 2044 and 2048 are leap years. Each normal year has 365 days i.e. $7 \times 52 + 1$ days. Each normal year shifts the date by one day. Further, each leap year shifts the date by another extra day. So, total extra days are $24 + 6 = 30 = 7 \times 4 + 2$. So, the net shift is of two days. Hence, 5th September 2050 is Monday.

14. Prove that

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots - \frac{1}{99} + \frac{1}{100} > \frac{1}{5}.$$

প্ৰমাণ কৰা যে

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots - \frac{1}{99} + \frac{1}{100} > \frac{1}{5}.$$

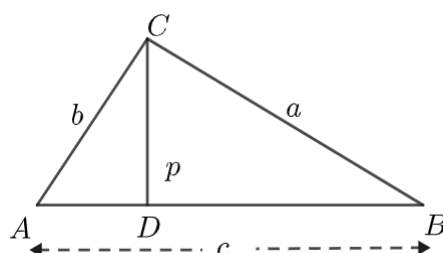
Ans : Group the terms in pairs as $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \dots + \left(\frac{1}{98} - \frac{1}{99}\right) + \frac{1}{100}$. The difference in each bracket is positive, because in every pair the first fraction has the smaller denominator; for example, $\frac{1}{2} - \frac{1}{3} = \frac{1}{2 \times 3} > 0$, $\frac{1}{4} - \frac{1}{5} = \frac{1}{4 \times 5} > 0$. Now, the sum of the first two pairs is $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) = \frac{1}{6} + \frac{1}{20} = \frac{10 + 3}{60} = \frac{13}{60}$. Since $\frac{1}{5} = \frac{12}{60}$, we have $\frac{13}{60} > \frac{1}{5}$. Adding the remaining positive terms makes the entire sum still larger. Hence,

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots - \frac{1}{99} + \frac{1}{100} > \frac{1}{5}.$$

15. $\triangle ABC$ is right angled at C . D is a point on AB such that CD is perpendicular to AB . Show that $\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}$.

$\triangle ABC$ ৰ C কোণটো হৈছে সমকোণ। D বিন্দুটো AB ৰ ওপৰত এনেভাৱে আছে যাতে AB ৰেখাখণ্ডৰ ওপৰত CD এডাল লম্ব। দেখুওৱা যে $\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}$.

Ans : We have the following diagram :



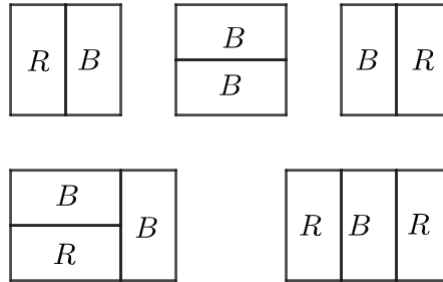
Let $BC = a, CA = b, AB = c$ and $CD = p$. In $\triangle ABC$, $AB^2 = AC^2 + BC^2$ i.e. $c^2 = b^2 + a^2$. Now observe that $\text{Area}(\triangle ABC) = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times AC \times BC$. So, $c \times p = b \times a$ i.e. $c = \frac{ab}{p}$. Then, $\left(\frac{ab}{p}\right)^2 = b^2 + a^2$ i.e. $\frac{a^2 b^2}{p^2} = b^2 + a^2$. Dividing both sides by $a^2 b^2$, we get $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ i.e. $\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}$.

16. You are given 2×1 rectangular tiles of red and blue colours, each having length 2 units and width 1 unit. You are allowed to place them side by side either horizontally or vertically to form bigger rectangular blocks. Some examples of 2×2 and 2×3 rectangular blocks are shown in the figure. Two blocks are different if they have different placing of tiles or different colour pattern of tiles.

- (a) How many different 2×2 rectangular blocks can you create ? 2
(b) How many different 2×3 rectangular blocks can you create ? 3
(c) How many different 2×4 rectangular blocks can you create ? 5

তোমাক কেইখনমান 2×1 আকাৰৰ ৰঙা আৰু নীলা ৰঙৰ আয়তাকৃতিৰ টাইলছ দিয়া হৈছে, যাৰ প্ৰতিখনৰ দৈৰ্ঘ্য 2 একক আৰু প্ৰস্থ 1 একক। তুমি সেই টাইলছবোৰক অনুভূমিকভাৱে এখনৰ কাষত এখনকৈ অথবা উলম্বভাৱে এখনৰ ওপৰত এখন কৈ ৰাখিব পাৰা আৰু তেনেকৈ ডাঙৰ আয়তাকৃতিৰ থূপ কিছুমান গঠন কৰিব পাৰা। 2×2 আকাৰৰ আৰু 2×3 আকাৰৰ কিছুমান ডাঙৰ আয়তাকৃতিৰ থূপৰ উদাহৰণ চিত্ৰত দিয়া হৈছে। দুটা থূপক পৃথক পৃথক বুলি কোৱা হ'ব যদিহে থূপ দুটাত থকা টাইলছসমূহৰ স্থান ভিন ভিন হয় অথবা টাইলছসমূহৰ ৰঙৰ আৰ্হি ভিন ভিন হয়।

- (a) তুমি তেনেকুৱা কিমানটা পৃথক 2×2 আকাৰৰ আয়তাকৃতিৰ থূপ গঠন কৰিব পাৰিবা? 2
(b) তুমি তেনেকুৱা কিমানটা পৃথক 2×3 আকাৰৰ আয়তাকৃতিৰ থূপ গঠন কৰিব পাৰিবা? 3
(c) তুমি তেনেকুৱা কিমানটা পৃথক 2×4 আকাৰৰ আয়তাকৃতিৰ থূপ গঠন কৰিব পাৰিবা? 5



Ans : There are two ways to place the tiles to form 2×2 rectangular blocks :



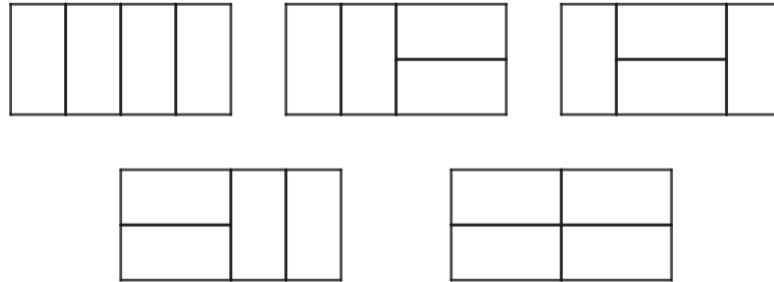
However, we also need to consider the colours of the tiles. Each tile can be either red or blue. For 2×2 blocks, we need two tiles. They can have $2 \times 2 = 4$ colour combinations i.e. RR, RB, BR, BB . So for the above two placement of tiles, there are exactly $2 \times 4 = 8$ different 2×2 rectangular blocks.

Next, there are three ways to place the tiles to form 2×3 rectangular blocks :



The three tiles in each block can have $2 \times 2 \times 2 = 8$ colour patterns i.e. $RRR, RRB, RBR, RBB, BRR, BRB, BBR, BBB$. So there are $3 \times 8 = 24$ different 2×3 rectangular blocks.

Again, there are five ways to place the tiles to form 2×4 rectangular blocks :



The four tiles in each block can have $2 \times 2 \times 2 \times 2 = 16$ colour patterns. So there are $5 \times 16 = 80$ different 2×4 rectangular blocks.

17. A student is counting the number of occurrences of the digit 5 in the natural numbers 1, 2, 3, ... For example, 5 appears once from 1 to 9, twice from 1 to 20, 21 times from 1 to 105 and so on. Find the smallest number n such that 5 occurs exactly 5000 times in the numbers from 1 to n .

এজন ছাত্ৰই 1, 2, 3, ... এই স্বাভাৱিক সংখ্যাবোৰত 5 অংকটো কিমানবাৰ থাকে সেয়া গণনা কৰি আছে। উদাহৰণস্বৰূপে, 1 ৰ পৰা 9 লৈ 5 অংকটো এবাৰ আছে, 1 ৰ পৰা 20 লৈ দুবাৰ আছে, 1 ৰ পৰা 105 লৈ মুঠ 21 বাৰ আছে ইত্যাদি। এনেকুৱা আটাইতকৈ সৰু সংখ্যা n টো বিচাৰি উলিওৱা যাতে 1 ৰ পৰা n লৈ সংখ্যাবোৰত 5 অংকটো মুঠ 5000 বাৰ থাকে।

Ans : Note that the number of occurrences of 5 from 1 to n is same as that from 0 to n . Consider all numbers from 0 to 9999. They can be written in the four digit format as 0000 to 9999. To find the number of times 5 occurs in the unit place, we consider the numbers of the form $***5$. Each of the remaining places can be filled in 10 ways. So, the number of times 5 appears as unit digit is $10 \times 10 \times 10 = 1000$. Number of times 5 occurs in the tens place is counted using numbers of the form $**5*$ and again there are $10 \times 10 \times 10 = 1000$ such numbers. Similarly, number of times 5 occurs in the hundreds place is also 1000 and number of times 5 occurs in the thousands place is also 1000. So from 1 to 9999, the digit 5 occurs exactly 4000 times. Next we consider numbers from 10000 onwards. Similar to above, number of times 5 occurs in the numbers of the form $10***$ is $100 + 100 + 100 = 300$. Number of times 5 occurs in the numbers of the form $11***$ is $100 + 100 + 100 = 300$ and number of times 5 occurs in the numbers of the form $12***$ is $100 + 100 + 100 = 300$. Thus we have got 4900 occurrences of 5 in the numbers from 1 to 12999. We need 100 more occurrences. If we consider numbers of the form $13***$ then the count will increase by 300. So, we consider numbers of the form $130**$ where 5 is counted $10 + 10 = 20$ times. Thus, numbers of the forms $130**, 131**, 132**, 133**$ and $134**$ will give the total count as $20 \times 5 = 100$. So, there are 5000 fives from 1 to 13499 but the smallest required n is 13495.

18. A triangular number counts the number of dots required to form a triangular arrangement in which each successive row contains one more dot than the row above it, starting with a single dot at the top. For example, the first few triangular numbers T_1, T_2, T_3, T_4 are shown in the figure. Find two consecutive triangular numbers whose sum is 7921.

এনেকুৱা এটা ত্ৰিভুজাকৃতিৰ সাজোন বিবেচনা কৰা যাৰ প্ৰথম শাৰীটোত এটা বিন্দু থাকে আৰু ইয়াৰ পৰৱৰ্তী প্ৰতিটো শাৰীত তাৰ ওপৰৰ শাৰীটোতকৈ এটা অধিক বিন্দু থাকে। এনেকুৱা ত্ৰিভুজাকৃতিৰ সাজোনসমূহত থকা মুঠ বিন্দুসমূহৰ সংখ্যাক ত্ৰিভুজীয় সংখ্যা বুলি কোৱা হয়। উদাৰণস্বৰূপে, প্ৰথম কেইটামান ত্ৰিভুজীয় সংখ্যা T_1, T_2, T_3, T_4 তলত দেখুওৱা হৈছে। এনেকুৱা দুটা ক্ৰমিক ত্ৰিভুজীয় সংখ্যা নিৰ্ণয় কৰা যাৰ যোগফল 7921।

$$\begin{array}{cccc}
 \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot \\
 & & \cdot & \cdot \\
 & & & \cdot \\
 1 & 3 = 1 + 2 & 6 = 1 + 2 + 3 & 10 = 1 + 2 + 3 + 4
 \end{array}$$

Ans : Observe that, the n th triangular number is the sum of the first n natural numbers. Further, observe the sums of two consecutive triangular numbers. $1 + 3 = 4, 3 + 6 = 9, 6 + 10 = 16, 10 + 15 = 25$. The sums are perfect squares. In fact it is always true. Let T_n and T_{n+1} be two consecutive triangular numbers. Then $T_n = 1 + 2 + 3 + \dots + n$ and $T_{n+1} = 1 + 2 + 3 + \dots + n + (n+1)$.

$$T_n + T_{n+1} = \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} = \frac{(n+1)(2n+2)}{2} = (n+1)^2 = 7921 = 89^2.$$

So, $n + 1 = 89$, so $n = 88$. Hence, $T_{88} = \frac{88 \times 89}{2} = 3916$, and $T_{89} = 3916 + 89 = 4005$. Therefore, the two consecutive triangular numbers are 3916 and 4005.