Assam Academy of Mathematics
Assam Mathematics Olympiad 2026
Category III (Classes IX - XI)
30th August 2026

Full marks : 100 Time : 3 hours

There are 18 questions. Questions 1 to 5 carry 2 marks each. Questions 6 to 13 carry 5 marks each. Questions 14 to 18 carry 10 marks each. Calculators or any other electric or digital devices are strictly prohibited.

ইয়াত 18 টা প্ৰশ্ন আছে। 1 ৰ পৰা 5 লৈ প্ৰতিটো প্ৰশ্নত 2 নম্বৰকৈ আছে। 6 ৰ পৰা 13 লৈ প্ৰতিটো প্ৰশ্নত 5 নম্বৰকৈ আছে। আৰু 14 ৰ পৰা 18 লৈ প্ৰতিটো প্ৰশ্নত 10 নম্বৰকৈ আছে। কেলকুলেটৰ বা অন্য কোনো বৈদ্যুতিক বা ডিজিটেল সঁজুলি সম্পূৰ্ণৰূপে নিষিদ্ধ।

There may be various other ways of solutions than those shown here. Queries or suggestions regarding the solutions can be mailed to mail@aamonline.in
ইয়াত দেখুওৱা ধৰণবিলাকৰ বাহিৰেও প্ৰশ্নবোৰৰ সমাধানৰ আন বিভিন্ন উপায় থাকিব পাৰে। সমাধানবোৰৰ বিষয়ে কিবা প্ৰশ্ন বা পৰামৰ্শ থাকিলে mail@aamonline.in লৈ মেইল কৰিব পাৰে।

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1. A student is writing all possible four digit "coloured numbers" using either red pen or blue pen for each digit. Two coloured numbers are distinct if either their numerical values are different or their colour patterns are different. How many distinct four digit coloured numbers exist ?

   Assamese version : এজন ছাত্ৰই এনেকুৱা চাৰিটা অংকবিশিষ্ট সকলোবোৰ সম্ভৱপৰ "ৰঙীণ সংখ্যা" লিখি আছে যাৰ প্ৰতিটো অংক ৰঙা অথবা নীলা কলমেৰে লিখা হয়। দুটা ৰঙীণ সংখ্যাক পৃথক পৃথক বুলি কোৱা হব যদিহে সিঁহতৰ সাংখ্যিক মান বেলেগ বেলেগ হয় অথবা সিঁহতৰ ৰঙৰ আৰ্হিসমূহ বেলেগ বেলেগ হয়। এনেকুৱা চাৰি অংকবিশিষ্ট কিমানটা পৃথক ৰঙীণ সংখ্যা থাকিব নিৰ্ণয় কৰা।

   Ans : Total number of four digit numbers is $9 \times 10 \times 10 \times 10 = 9000$. Each of these numbers can be coloured in $2 \times 2 \times 2 \times 2 = 16$ ways. Thus, required number of distinct four digit coloured numbers is $9000 \times 16 = 144000$.

2. A student claims to have found four consecutive positive integers whose product is 49444944. Justify if the claim is true or false ?

   এজন শিক্ষার্থীয়ে দাবী কৰিছে যে তেওঁ এনে চাৰিটা ক্ৰমাগত ধনাত্মক পূৰ্ণসংখ্যা বিচাৰি পাইছে যিবোৰৰ গুণফল 49444944। এই দাবীটো সত্য নে মিছা, যুক্তিসহকাৰে প্ৰমাণ কৰক। / এগৰাকী শিক্ষার্থীয়ে এনেকুৱা চাৰিটা ক্ৰমিক অখণ্ড ধনাত্মক সংখ্যা বিচাৰি পোৱা বুলি দাবী কৰিছে যাৰ পূৰণফল 49444944। তেওঁৰ দাবীটোৰ সত্যাসত্যতা নিৰূপণ কৰা।

   Ans : The product of four consecutive integers is always divisible by 24. However, a multiple of 24 may not be a product of four consecutive integers. Here, $49444944 = 4944 \times 10001 = 3 \times 1648 \times 10001 = 3 \times 8 \times 206 \times 10001$. So it is divisible by 24. No conclusion can be drawn from here. Observe that if $n, n+1, n+2, n+3$ are four consecutive integers then, $n(n+1)(n+2)(n+3) = (n^2+3n)(n^2+3n+2) = (n^2+3n)^2 + 2(n^2+3n) = (n^2+3n+1)^2 - 1$ which is 1 less than a perfect square. But, $49444944 + 1 = 49444945$ is not a perfect square as the last two digits are 45. Thus, the claim is false.

3. A play arena for kids is being filled with balls of red, blue and green colours. What is the smallest number of balls that should be placed in the arena, without looking at the colours, in order to guarantee that either there are at least 21 red balls or at least 31 blue balls or at least 26 green balls ?

   Assamese version : শিশু ক্ৰীড়াংগন এখন ৰঙা, নীলা আৰু সেউজীয়া ৰঙৰ বলেৰে সজাই তোলা হৈছে। বলৰ ৰঙসমূহ নোচোৱাকৈ কেইটামান বল তাতে ৰখা হ'ল। ন্যূনতম কিমানটা বল ক্ৰীড়াংগনখনত ৰাখিলে এয়া নিশ্চিত হ'ব যে তাত অন্ততঃ 21 টা ৰঙা বল অথবা অন্ততঃ 31 টা নীলা বল অথবা অন্ততঃ 26 টা সেউজীয়া বল থাকিব?

Ans : Note that we cannot count the balls as per colour. We can only count the total number of balls that is being placed and make sure that the requirement is satisfied. In the worst possible situation it may happen that we end up in placing 20 red balls, 30 blue balls and 25 red balls. However, if we place $20 + 30 + 25 + 1 = 76$ balls, then either there are at least 21 red balls or at least 31 blue balls or at least 26 green balls. Placing anything less than 76 balls doesn't guarantee the necessary requirement. Thus, 76 is the required minimum number of balls.

4. Let a, b, c be integers and suppose that all the roots of the equation $x^3 + ax^2 + bx + c = 0$ are integers. If the sum of the roots is 2026 and the absolute value of their product is also 2026, find all possible roots.

   ধৰা হ'ল, a,b,c তিনিটা অখণ্ড সংখ্যা আৰু $x^3 + ax^2 + bx + c = 0$ সমীকৰণটোৰ সকলো মূল অখণ্ড সংখ্যা। যদি মূলসমূহৰ যোগফল 2026 আৰু মূলসমূহৰ পূৰণফলৰ পৰম মানো 2026 হয়, তেন্তে সম্ভৱপৰ সকলো মূল নির্ণয় কৰা।

   Ans : The equation has three roots. Since we are considering integral roots only and the absolute value of the product of roots is 2026, observe that $2026 = 1 \times 2 \times 1013 = 1 \times 1 \times 2026$. Sum of roots can be 2026 only when one of the roots is 2026 and the other two roots are 1 and -1.

5. If $a_1, a_2, \ldots, a_{10}$ are roots of the equation $x^{10} + x^2 + 1 = 0$ show that

   $(1 + |a_1|)(1 + |a_2|) \ldots (1 + |a_{10}|) > 1021$.

   Assamese Version: যদি $a_1, a_2, \ldots, a_{10}$ এইকেইটা $x^{10} + x^2 + 1 = 0$ সমীকৰণটোৰ মূল হয়, তেন্তে দেখুওৱা যে $(1 + |a_1|)(1 + |a_2|) \ldots (1 + |a_{10}|) > 1021$.

   Ans : Product of the roots, $a_1 a_2 \ldots a_{10} = 1$. So none of the roots is 0. Thus, we have

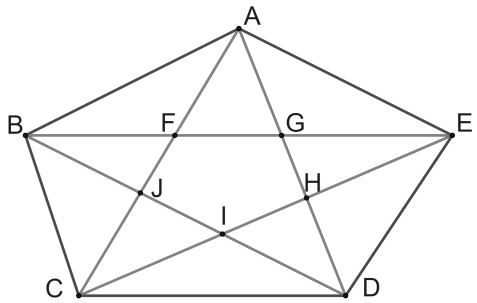
   $|a_1| > 0, |a_2| > 0, \ldots, |a_{10}| > 0$ and $|a_1| \cdot |a_2| \cdots |a_{10}| = 1$.

   By $AM - GM$ inequality, $\frac{1 + |a_1|}{2} \geq \sqrt{1 \cdot |a_1|}, \frac{1 + |a_2|}{2} \geq \sqrt{1 \cdot |a_2|}, \ldots, \frac{1 + |a_{10}|}{2} \geq \sqrt{1 \cdot |a_{10}|}$ so that

   $(1 + |a_1|)(1 + |a_2|) \ldots (1 + |a_{10}|) \geq 2^{10} \times \sqrt{|a_1| \cdot |a_2| \cdots |a_{10}|} = 1024 > 1021$.

6. Can there be a convex pentagon such that the sum of the lengths of its diagonals is 50 cm and the perimeter is 53 cm ? Justify your answer.

   Assamese Version : এনেকুৱা এটা উত্তল পঞ্চভুজ থাকিব পাৰেনে যাৰ কর্ণসমূহৰ দৈর্ঘ্যৰ যোগফল 50 চেমি আৰু পৰিসীমা 53 চেমি? তোমাৰ উত্তৰটোৰ যুক্তিযুক্ততা বিচাৰ কৰা।



   Ans : There cannot be such a convex pentagon. We can show that the sum of the diagonals of a convex pentagon is greater than its perimeter. Using triangle equality, $FA + FB > AB, JB + JC > BC, IC + ID > CD, HD + HE > DE$ and $GE + GA > EA$. Adding,

   $FA + FB + JB + JC + IC + ID + HD + HE + GE + GA > AB + BC + CD + DE + EA$

Observe that the sum of the diagonals of the pentagon is greater than the sum on the left and the sum on the right is the perimeter of the pentagon $ABCDE$. Hence, the sum of the diagonals of a convex pentagon is greater than its perimeter.

7. Show that 1280000401 is a composite number.

   প্রমাণ কৰক যে 1280000401 এটা যৌগিক সংখ্যা । / দেখুওৱা যে 1280000401 সংখ্যাটো এটা যৌগিক সংখ্যা।

   Ans : We have $1280000401 = 1280000000 + 400 + 1 = 20^7 + 20^2 + 1$. Let $x = 20$. We try to factorize $x^7 + x^2 + 1$. We have, $x^7 + x^2 + 1 = x^7 - x + x^2 + x + 1 = x(x^6 - 1) + (x^2 + x + 1) = x(x^3 - 1)(x^3 + 1) + (x^2 + x + 1) = (x^2 + x + 1)\{x(x - 1)(x^3 + 1) + 1\}$. Thus, one factor is $x^2 + x + 1 = 20^2 + 20 + 1 = 421$. Hence the number is composite.

8. Can you find integers x, y, z such that $x^3 - xyz = -1, y^3 - xyz = -2$ and $z^3 - xyz = -15$? Justify your answer.

   Assamese version : এনেকুৱা তিনিটা অখণ্ড সংখ্যা x, y, z বিচাৰি উলিয়াব পৰা যাবনে যাতে $x^3 - xyz = -1, y^3 - xyz = -2$ আৰু $z^3 - xyz = -15$? তোমাৰ সমাধানটোৰ যুক্তিযুক্ততা বিচাৰ কৰা।

   Ans : Let $xyz = a$. Then $x^3 = a - 1, y^3 = a - 2, z^3 = a - 15$. So, $x^3y^3z^3 = (a-1)(a-2)(a-15)$ i.e., $a^3 = (a-1)(a^2-17a+30)$ i.e., $a^3 = a^3 - 17a^2 + 30a - a^2 + 17a - 30$ i.e., $18a^2 - 47a + 30 = 0$. Solving, $18a^2 - 27a - 20a + 30 = 0$ i.e., $(2a - 3)(9a - 10) = 0$ i.e., $a = \dfrac{3}{2}$ or $a = \dfrac{10}{9}$. Thus, the product xyz is not an integer. Hence, there doesn't exist integers x, y, z satisfying the given condition.

9. How many three digit positive integers are there such that the three digits of every integer, taken from left to right, form an arithmetic progression ?

   তিনিটা অংকবিশিষ্ট এনেকুৱা কিমানটা ধনাত্মক অখণ্ড সংখ্যা পোৱা যাব যাতে প্ৰতিটো অখণ্ড সংখ্যাৰ তিনিওটা অংকই, বাওঁফালৰ পৰা সোঁফাললৈ, একোটা সমান্তৰ অনুক্ৰম গঠন কৰে?

   Ans : Since the digits form an arithmetic progression, so the common difference d can only take values $0, \pm1, \pm2, \pm3, \pm4$. Observe that if $d = 5$, then we cannot find three digits with common difference 5. For example, if the first digit is 1, then the second digit has to be 6 and then the third digit 11 but 11 is not a digit. Thus, there are the following cases :

   (a) $d = 0$ gives 9 such numbers $111, 222, 333, 444, 555, 666, 777, 888, 999$.

   (b) $d = \pm1$ gives $123, 234, 345, 456, 567, 678, 789$ and their reverse numbers. Further, observe that 210 is also to be counted. There are 15 such numbers.

   (c) $d = \pm2$ gives $135, 246, 357, 468, 579$ and their reverse. Further, 420 is also to be counted. There are 11 such numbers.

   (d) $d = \pm3$ gives $147, 258, 369$ and their reverse. Further, 630 is also to be counted. There are 7 such numbers.

   (e) $d = \pm4$ gives 159, 951 and 840. So, 3 such numbers.

   Thus, there are $9 + 15 + 11 + 7 + 3 = 45$ such numbers.

10. Let $p(x) = -x^3 + ax^2 + bx + c$ be a polynomial where a, b, c are odd integers. Let p_i denote the value of $p(x)$ at $x = i$. If $(p_1 + p_2 + p_3)(p_1p_2 + p_2p_3 + p_3p_1) = p_1p_2p_3$, find the value of $3p_2 - 2p_1 + p_0$.

    Assamese version : ধৰা হ'ল, $p(x) = -x^3 + ax^2 + bx + c$ এটা বহুপদ যাতে a, b, c তিনিটা অযুগ্ম অখণ্ড সংখ্যা। ধৰা হ'ল, p_i য়ে $x = i$ ত $p(x)$ ৰ মানটোক সূচায়। যদি $(p_1+p_2+p_3)(p_1p_2+p_2p_3+p_3p_1) = p_1p_2p_3$, তেন্তে $3p_2 - 2p_1 + p_0$ ৰ মান নিৰ্ণয় কৰা।

Ans : Using the algebraic identity, $(x+y+z)(xy+yz+zx)-xyz=(x+y)(y+z)(z+x)$,

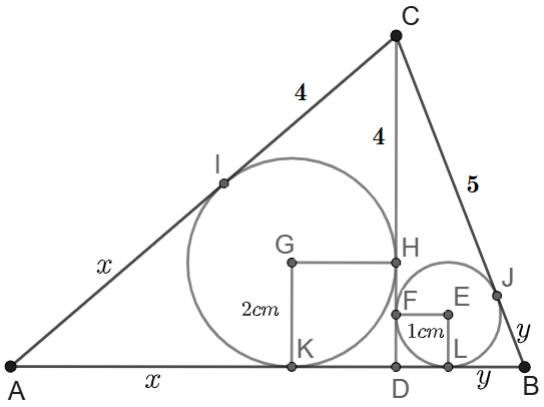
$$(p_1+p_2+p_3)(p_1p_2+p_2p_3+p_3p_1)-p_1p_2p_3=0$$
$$\Rightarrow (p_1+p_2)(p_2+p_3)(p_3+p_1)=0$$

Now, $p_1=-1+a+b+c, p_2=-8+4a+2b+c, p_3=-27+9a+3b+c$. Thus, $p_1+p_2=-9+5a+3b+2c$ which is odd as a,b,c are odd. So, $p_1+p_2\neq 0$. Next, $p_2+p_3=-35+13a+5b+2c$ which is odd and so $p_2+p_3\neq 0$. Also, $p_3+p_1=-28+10a+4b+2c$ which is even and by the above condition, $p_3+p_1=0$ i.e. $10a+4b+2c=28$.

$$\begin{aligned} 3p_2-2p_1+p_0 &= -24+12a+6b+3c-(-2+2a+2b+2c)+c \\ &= 10a+4b+2c-22 \\ &= 28-22=6 \end{aligned}$$

11. In ΔABC, D is a point on AB such that $CD\perp AB$, $CD=6$ cm and the radii of the incircles of ΔACD and ΔBCD are 2 cm and 1 cm respectively. Find the area of ΔABC.

ΔABC ত D হৈছে AB ৰ ওপৰত থকা এটা বিন্দু যাতে $CD\perp AB$, $CD=6$ চেমি আৰু ΔACD আৰু ΔBCD ৰ অন্তর্বৃত্ত দুটাৰ ব্যসার্ধ সমূহ হৈছে যথাক্রমে 2 চেমি আৰু 1 চেমি। ΔABC ৰ কালি নির্ণয় কৰা।



Ans : We construct the figure and label the points. The sides of $\triangle ACD$ and $\triangle BCD$ are tangents to their respective incircles. Since lengths of tangents from an external point are equal, so let $AK=AI=x, BL=BJ=y$. Also, given $CD=6$. So, $CH=CI=6-2=4$ and $CF=CJ=4+1=5$. Then in $\triangle ACD$, $AC=x+4, AD=x+2$ and in $\triangle BCD$, $BC=y+5, BD=y+1$. By Pythagoras theorem, $(x+4)^2=(x+2)^2+6^2$ which gives $8x+16=4x+4+36$ i.e. $4x=24$ i.e. $x=6$. Also, $(y+5)^2=6^2+(y+1)^2$ which gives $10y+25=36+2y+1$ i.e. $8y=12$ i.e. $y=\frac{3}{2}$. Thus, area of $\triangle ABC=\frac{1}{2}AB\times CD=\frac{1}{2}\times(x+2+1+y)\times 6=(9+\frac{3}{2})\times 3=\frac{63}{2}=31.5$ sq. cm.

12. Find a four digit number of the form $abcd$ such that $4\times abcd=dcba$. How many such four digit numbers are there ?

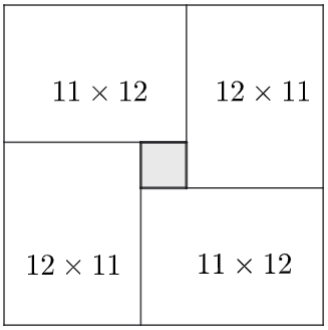
Assamese version : এনেকুৰা এটা চাৰি অংকবিশিষ্ট সংখ্যা $abcd$ নির্ণয় কৰা যাতে $4\times abcd=dcba$। এনেকুৰা মুঠ কিমানটা চাৰি অংকবিশিষ্ট সংখ্যা থাকিব?

Ans : Given, $4\times abcd=dcba$. So, $dcba$ is even i.e. a is even. Also, $0<a<3$ as $4\times 3000=12000$ which is of five digits. Thus, the only possibility is $a=2$. Now we have $2bcd\times 4=dcb2$. Since $2bcd\geq 2000$, so $dcb2\geq 8000$. Hence, $d\geq 8$. But $4\times d$ has unit digit 2. So, the only possibility is $d=8$. So, $2bc8\times 4=8cb2$ i.e. $(2000+100b+10c+8)\times 4=8000+100c+10b+2$ i.e. $390b+30=60c$ i.e. $13b+1=2c$. So, b must be odd. Now, c is a digit. So, $2c\leq 18$. Hence, $13b+1\leq 18$ i.e. $13b\leq 17$ but again b is an odd digit. So, $13b\geq 13$. Thus, the only possibility is $b=1$. This gives $c=7$. Hence, the required number is 2178 and it is the only such four digit number.

13. A 23×23 square floor is completely tiled by $1 \times 1, 2 \times 2, 3 \times 3$ tiles. What is the minimum number of 1×1 tiles required for this ?

Assamese version : এখন 23×23 আকাৰৰ বৰ্গাকৃতিৰ মজিয়া $1 \times 1, 2 \times 2, 3 \times 3$ আকাৰৰ বৰ্গাকৃতিৰ টাইলচেৰে সম্পূৰ্ণৰূপে সজাই তোলা হ'ল। এই কামটোৰ বাবে ন্যূনতম কিমানখন 1×1 আকাৰৰ টাইলচ লাগিব?

Ans : Suppose that no 1×1 tiles are used. Imagine colouring the rows of the 23×23 floor alternately black and white, starting with the first row in black. Then there are 12 black rows and 11 white rows. Also, there will be 23 more black unit squares than white unit squares. Further, observe that each 3×3 tile will cover exactly three more unit squares of one colour than the other but each 2×2 tile covers equal number of unit squares of each colour. Thus, the difference between the number of black unit squares and white unit squares must be a multiple of 3, no matter how one places the tiles but the difference between the number of black unit squares and white unit squares is exactly 23 which is not a multiple of 3. So, our assumption is false. Thus, at least one 1×1 tile is required.



Next, we try to see if only one 1×1 tile is sufficient. Consider the 1×1 tile at the centre. Then, the floor can be divided into two 12×11 portions and two 11×12 portions. The 12×11 portion can be split into two parts as 12×3 and 12×8. The first part can be tiled using four 3×3 tiles and the second part can be tiled by twenty four 2×2 tiles. Similarly the 11×12 portion can also be covered only by 2×2 and 3×3 tiles.

14. Find all real values of p for which the equation $\sqrt{x^2-p}+2\sqrt{x^2-1}=x$ has real solutions.

Assamese version : p ৰ আটাইবোৰ বাস্তৱ মান নিৰ্ণয় কৰা যাৰ বাবে $\sqrt{x^2-p}+2\sqrt{x^2-1}=x$ সমীকৰণটোৰ বাস্তৱ সমাধান থাকে।

Ans : Given, $\sqrt{x^2-p}+2\sqrt{x^2-1}=x$. Thus, $x^2-p \geq 0$ and $x^2-1 \geq 0$ i.e. $x^2 \geq p$ and $x^2 \geq 1$. Squaring the equation, we get $4\sqrt{x^2-p}\sqrt{x^2-1}=p+4-4x^2$. Since the LHS is non-negative, so $p+4-4x^2 \geq 0$ i.e. $x^2 \leq \dfrac{p+4}{4}$. Again squaring the equation, we get $16(x^2-p)(x^2-1)=(p+4-4x^2)^2$ i.e. $8(2-p)x^2=(p-4)^2$. Since $x^2 \geq 0$, so $2-p \geq 0$. Clearly, $p \neq 2$ as otherwise we will get $0=4$. Hence, $p<2$. So, $x^2=\dfrac{(p-4)^2}{8(2-p)}$. We now check the possible values of p for which the previous restrictions on x are true.

$x^2 \geq 1 \Rightarrow \dfrac{(p-4)^2}{8(2-p)}-1 \geq 0 \Rightarrow \dfrac{p^2}{8(2-p)} \geq 0 \Rightarrow 2-p>0 \Rightarrow p<2.$

$x^2 \geq p \Rightarrow \dfrac{(p-4)^2}{8(2-p)} \geq p \Rightarrow \dfrac{9p^2-24p+16}{8(2-p)} \geq 0 \Rightarrow \dfrac{(3p-4)^2}{8(2-p)} \geq 0 \Rightarrow 2-p>0 \Rightarrow p<2.$

$x^2 \leq \dfrac{p+4}{4} \Rightarrow \dfrac{(p-4)^2}{8(2-p)} \leq \dfrac{p+4}{4} \Rightarrow \dfrac{(p-4)^2-2(p+4)(2-p)}{8(2-p)} \leq 0 \Rightarrow \dfrac{3p^2-4p}{8(2-p)} \leq 0$ which gives $3p^2-4p \leq 0$ as $p<2$. So, $p(3p-4) \leq 0$ i.e. $0 \leq p \leq \dfrac{4}{3}$. Note that $\dfrac{4}{3}<2$.

Thus, the common values of p which satisfy all the constraints are $0 \leq p \leq \frac{4}{3}$.

15. In a finite sequence of real numbers, the sum of any consecutive 5 terms is negative and the sum of any consecutive 7 terms is positive. What is the maximum possible number of terms of such a sequence. Construct such a sequence of maximum possible terms.

বাস্তৱ সংখ্যাৰ সসীম অনুক্রম এটাত, যিকোনো ক্রমিক 5 টা পদৰ যোগফল ঋণাত্মক আৰু যিকোনো ক্রমিক 7 টা পদৰ যোগফল ধনাত্মক। এনেকুৱা এটা অনুক্রমত থাকিব পৰা সম্ভৱপৰ সর্বোচ্চ পদৰ সংখ্যা নির্ণয় কৰা। তেনেকুৱা সর্বোচ্চ সংখ্যক পদযুক্ত এটা অনুক্রম লিখি উলিওৱা।

Ans : Let $a_1, a_2, a_3, \ldots$ be the sequence. Let us consider the following table :

$$\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_2 & a_3 & a_4 & a_5 & a_6 \\ a_3 & a_4 & a_5 & a_6 & a_7 \\ a_4 & a_5 & a_6 & a_7 & a_8 \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_7 & a_8 & a_9 & a_{10} & a_{11} \end{array}$$

Observe that the rows can give all possible sums of consecutive 5 terms upto a_{11} and the columns can give all possible sums of consecutive 7 terms upto a_{11}. By the given condition, each row sum is negative and each column sum is positive. So, the total row wise sum of the table is negative but the total column wise sum of the same table is positive. This is not correct. Hence, such a sequence cannot contain more than 10 terms.

Next, we try to see if we can actually construct such a sequence of 10 terms. we consider the partial sums $s_1 = a_1, s_2 = a_1 + a_2, s_3 = a_1 + a_2 + a_3$ and so on. By the given condition, $s_5 < 0, s_6 - s_1 < 0, s_7 - s_2 < 0, s_8 - s_3 < 0, s_9 - s_4 < 0, s_{10} - s_5 < 0$ i.e.

$s_5 < 0, s_6 < s_1, s_7 < s_2, s_8 < s_3, s_9 < s_4, s_{10} < s_5$.

Further, $s_7 > 0, s_8 - s_1 > 0, s_9 - s_2 > 0, s_{10} - s_3 > 0$ i.e. $s_7 > 0, s_8 > s_1, s_9 > s_2, s_{10} > s_3$. We combine the inequalities to form a chain of inequalities. We have,

$s_4 > s_9 > s_2 > s_7 > 0$ and $s_6 < s_1 < s_8 < s_3 < s_{10} < s_5 < 0$. We now give arbitrary values to the partial sums so that they satisfy both the inequality chains. Then we can compute the terms of the sequence.

Let $s_4 = 6, s_9 = 5, s_2 = 4, s_7 = 3$ and $s_6 = -6, s_1 = -5, s_8 = -4, s_3 = -3, s_{10} = -2, s_5 = -1$. In order to compute the terms, it is better to write the partial sums in a tabular form :

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
-5	4	-3	6	-1	-6	3	-4	5	-2

Now, $a_1 = -5, a_2 = s_2 - s_1 = 9, a_3 = s_3 - s_2 = -7, a_4 = s_4 - s_3 = 9$ and so on. Thus, one such sequence is $-5, 9, -7, 9, -7, -5, 9, -7, 9, -7$. In fact we can construct as many such sequences as we want just by changing the values of the partial sums with due respect to the inequality chains.

16. In $\triangle ABC$, we have $AB = AC$. A circle is tangent internally to the circumcircle of the triangle and also to the sides AB and AC at T, P, Q respectively. Prove that

(a) A and T are collinear with the centres of the two circles.

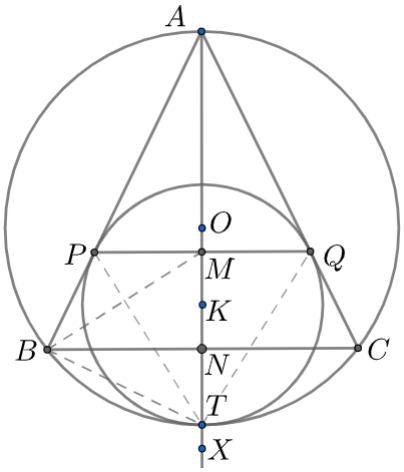
(b) the mid-point of PQ is the centre of the incircle of $\triangle ABC$.

Assamese version : $\triangle ABC$ ৰ $AB = AC$। এই ত্রিভুজটোৰ পৰিবৃত্তটোক আৰু AB আৰু AC বাহু দুটাক আন এটা বৃত্তই অভ্যন্তৰীণভাৱে ক্রমে T, P আৰু Q বিন্দুত স্পর্শ কৰি আছে।

প্রমাণ কৰা যে

(a) A আৰু T বিন্দু দুটা আগৰ বৃত্ত দুটাৰ কেন্দ্ৰৰ সৈতে একেৰেখীয়।

(b) PQ ৰেখাখণ্ডৰ মধ্যবিন্দুটোৱেই $\triangle ABC$'ৰ অন্তর্বৃত্তৰ কেন্দ্ৰ।



Ans : We have AP and AQ are lengths of tangents to the smaller circle from point A. Hence, $AP = AQ$. Also, since $AB = AC$, so $PB = QC$. Thus, $\frac{AP}{PB} = \frac{AQ}{QC}$, so that $PQ||BC$ by converse of basic proportionality theorem. Let AX be the axis of symmetry of $\triangle ABC$. Since $\triangle ABC$ is isosceles, AX cuts PQ at its mid-point M and also cuts BC at its mid-point N. Further, the circumcentre O of $\triangle ABC$ lies on AX. Again, AP and AQ are tangents to the smaller circle from point A. So, the centre K of the smaller circle also lies on the axis of symmetry AX. Thus, A, M, N, O, K are collinear. Now, the circles touch internally at T. Hence, T and the centres O, K of the circles are collinear. It follows that A, M, N, O, K, T are collinear and all lie on the axis of symmetry of $\triangle ABC$. Thus, AT is a diameter of the circumcircle of $\triangle ABC$.

Now, AM bisects $\angle BAC$. We join BM and try to show that BM bisects $\angle ABC = \alpha$. Since $PQ||BC$, so $\angle APQ = \angle ABC = \alpha$. Also, AP is tangent and PQ is a chord. So, $\angle APQ = \angle PTQ = \alpha$. Join BT. Since AT is diameter, so $\angle ABT = 90^0$. Also, $AT \perp PQ$ so that $\angle PMT = 90^0$. Thus, $PBMT$ is a cyclic quadrilateral. So, $\angle PBM = \angle PTM$. Observe that $PT = QT$ as $\triangle PTM \cong \triangle QTM$. Thus, TM bisects $\angle PTQ$. So, $\angle PTM = \frac{\alpha}{2}$ which gives $\angle PBM = \frac{\alpha}{2} = \frac{1}{2}\angle ABC$. So, BM bisects $\angle ABC$. Hence, M is the incentre of $\triangle ABC$.

17. For $n \in \mathbf{N}$, let $p(n)$ denote the product of the digits of n. Consider the numbers n having at most three digits and all digits prime. How many of these have the property $p(p(n)) \neq 0$?

ধৰা হ'ল, যিকোনো $n \in \mathbf{N}$ ৰ বাবে, $p(n)$ য়ে n ৰ অংককেইটাৰ পূৰণফলক সূচায়। এনেকুৱা আটাইবোৰ সংখ্যা n বিবেচনা কৰা যাৰ অতি বেছি তিনিটা অংক থাকে আৰু আটাইকেইটা অংক মৌলিক। ইয়াৰে কিমানটা সংখ্যাই $p(p(n)) \neq 0$ এই চর্তটো সিদ্ধ কৰে?

Ans : Since n has all prime digits, so the only possible digits of n are $2, 3, 5, 7$. Thus, $p(p(n)) \neq 0$ implies that n cannot have both 2 and 5 appearing as its digits. However, even if 2 and 5 do not appear, $p(n)$ can have an internal 0. This is the case when $3, 5, 7$ are the digits, so that $p(n) = 105$ that gives $p(p(n)) = 0$. So, we should count only those n which have digits among $2, 3, 7$ or among $3, 5, 7$, excluding the $3! = 6$ three digit numbers formed with $3, 5, 7$.

If n is of single digit, then all four options $2, 3, 5, 7$ satisfy $p(p(n)) \neq 0$.

If n is of two digits, $3 \times 3 = 9$ numbers have digits among $2, 3, 7$ and $3 \times 3 = 9$ numbers have digits among $3, 5, 7$. But, the numbers having digits among $3, 7$ are counted in both. There are $2 \times 2 = 4$ such numbers. So, there are $9 + 9 - 4 = 14$ two digit numbers that satisfy $p(p(n)) \neq 0$.

If n is of three digits, $3 \times 3 \times 3 = 27$ numbers have digits among $2, 3, 7$ and 27 numbers have digits among $3, 5, 7$. But $2 \times 2 \times 2 = 8$ numbers have digits among $3, 7$ and are counted in both. Also, there are $3! = 6$ numbers having digits $3, 5, 7$ for which $p(p(n)) = 0$. So, there are $27 + 27 - 8 - 6 = 40$ three digit numbers that satisfy $p(p(n)) \neq 0$.

Hence, there are $4 + 14 + 40 = 58$ numbers of at most three digits that satisfy $p(p(n)) \neq 0$.

18. Consider functions $f : \mathbb{N} \to \mathbb{N}$ such that $f(n)|f(m)$ whenever $n|m$ and $f(n)$ is prime whenever n is composite. Given that $gcd(f(61), f(74)) = 1$, find $f(6174)$ when f is such that $\sum_{d|6174} f(d)$ is of four digits and the least possible. The summation runs over all the positive divisors of 6174.

এনেকুৰা আটাইবোৰ ফলন $f : \mathbb{N} \to \mathbb{N}$ বিবেচনা কৰা যাতে $n|m$ হ'লে $f(n)|f(m)$ হয় আৰু n এটা যৌগিক সংখ্যা হ'লে $f(n)$ এটা মৌলিক সংখ্যা হয়। দিয়া আছে, গসাউ$(f(61), f(74)) = 1$ । তেন্তে $f(6174)$ ৰ মান নির্ণয় কৰা যাতে $\sum_{d|6174} f(d)$ এই যোগফলটো চাৰি অংকবিশিষ্ট আৰু সর্বনিম্ন হয়। মন কৰিবা এই যোগফলটো 6174 ৰ আটাইবোৰ ধনাত্মক উৎপাদকৰ ওপৰত বিবেচনা কৰা হৈছে।

Ans : Observe that $f(1) = 1$. Otherwise, if $f(1) = m$ where $m > 1$, then $1|n \Rightarrow m|f(n)$ for every $n \in \mathbb{N}$, which contradicts $gcd(f(61), f(74)) = 1$. Now, $6174 = 2 \times 3^2 \times 7^3$. It has 24 divisors. Out of these, only 4 are non-composite i.e. 1,2,3,7. Let $f(6174) = p$ where p is prime. Then, $d|6174 \Rightarrow f(d)|p$. So, $f(d)$ is 1 or p for any divisor of 6174. Further, $f(d) = p$ for every composite divisor of 6174. There are 20 composite divisors. So,

$$S = \sum_{d|6174} f(d) = 20p + f(1) + f(2) + f(3) + f(7) = 20p + 1 + f(2) + f(3) + f(7).$$

Now, we need to look for the least possible four digit value of $\sum_{d|6174} f(d)$. If $f(2) = f(3) = f(7) = 1$, then $20p + 4 \geq 1000 \Rightarrow p \geq 49$. Since p is prime, so $p = 53$. Hence, $S = 1064$. If exactly one of $f(2), f(3), f(7)$ is p, then $21p + 3 \geq 1000 \Rightarrow p > 47$. The next higher prime 53 will give $S = 1116$ which is much higher and so not desirable. If exactly two of $f(2), f(3), f(7)$ are p, then $22p + 2 \geq 1000 \Rightarrow p > 45$. Thus, $p = 47$ gives $S = 1036$ in this case. Finally, if $f(2) = f(3) = f(7) = p$, then $23p + 1 \geq 1000 \Rightarrow p > 43$. So $p = 47$ gives $S = 1082$. Hence, the least possible four digit value of S is $22 \times 47 + 2 = 1036$ and is obtained when exactly two of $f(2), f(3), f(7)$ are 47. One such f is given by $f(1) = f(2) = f(61) = 1$ and $f(n) = 47$ for every other n. So, under the given conditions $f(6174) = 47$.