

**Assam Academy of Mathematics**  
**Assam Mathematics Olympiad 2025**  
**Category II (Classes VII - VIII)**  
**24th August 2025**

Full marks : 100

Time : 3 hours

There are 18 questions. Questions 1 to 5 carry 2 marks each. Questions 6 to 13 carry 5 marks each. Questions 14 to 18 carry 10 marks each.

ইয়াত 18 টা প্ৰশ্ন আছে। 1 ৰ পৰা 5 লৈ প্ৰতিটো প্ৰশ্নত 2 নম্বৰকৈ আছে। 6 ৰ পৰা 13 লৈ প্ৰতিটো প্ৰশ্নত 5 নম্বৰকৈ আছে। আৰু 14 ৰ পৰা 18 লৈ প্ৰতিটো প্ৰশ্নত 10 নম্বৰকৈ আছে।

There may be various other ways of solutions than those shown here. Queries or suggestions regarding the solutions can be mailed to [mail@aamonline.in](mailto:mail@aamonline.in)

ইয়াত দেখুওৱা ধৰণবিলাকৰ বাহিৰেও প্ৰশ্নবোৰৰ সমাধানৰ আন বিভিন্ন উপায় থাকিব পাৰে। সমাধানবোৰৰ বিষয়ে কিবা প্ৰশ্ন বা পৰামৰ্শ থাকিলে [mail@aamonline.in](mailto:mail@aamonline.in) লৈ মেইল কৰিব পাৰে।

1. In a quiz, 5 marks are awarded for each correct answer but 3 marks are deducted for every incorrect answer. A student answered 9 questions correctly and she was awarded a total of 24 marks. How many questions did she answer incorrectly ?

এখন কুইজ প্ৰতিযোগিতাত প্ৰতিটো শুদ্ধ উত্তৰ দিয়াৰ বাবে 5 নম্বৰ দিয়া হয় আৰু অশুদ্ধ উত্তৰৰ বাবে 3 নম্বৰ বিয়োগ কৰা হয়। যদি, এগৰাকী প্ৰতিযোগীয়ে 9 টা প্ৰশ্নৰ শুদ্ধ উত্তৰ কৰি সৰ্বমুঠ 24 নম্বৰ লাভ কৰে তেন্তে তেওঁৰ কিমানটা প্ৰশ্নৰ উত্তৰ অশুদ্ধ হ'ল ?

Ans : Marks for correct answer is  $9 \times 5 = 45$ . Total marks obtained is 24. Hence, total marks for incorrect answers is  $24 - 45 = -21$ . So, number of incorrect answers is  $\frac{-21}{-3} = 7$ .

2. Find the smallest perfect square that is divisible by each of the numbers 14, 15 and 16.

14, 15 আৰু 16 ৰে বিভাজ্য আটাইতকৈ ক্ষুদ্ৰতম পূৰ্ণ বৰ্গ সংখ্যাটো নিৰ্ণয় কৰা।

Ans :  $14 = 2 \times 7$ ,  $15 = 3 \times 5$ ,  $16 = 2 \times 2 \times 2 \times 2$ . So, LCM of 14, 15 and 16 is  $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 4^2 \times 3 \times 5 \times 7$ . The smallest perfect square that is divisible by each of the numbers 14, 15 and 16 is same as the smallest perfect divisible by  $4^2 \times 3 \times 5 \times 7$  which is  $4^2 \times 3^2 \times 5^2 \times 7^2 = 4 \times 4 \times 9 \times 25 \times 49 = 4 \times 441 \times 100 = 176400$ .

3. Find the value of  $(256)^{0.16} \times (16)^{0.18}$ .

মান নিৰ্ণয় কৰা :  $(256)^{0.16} \times (16)^{0.18}$

Ans :  $(256)^{0.16} \times (16)^{0.18} = (2^8)^{0.16} \times (2^4)^{0.18} = 2^{8 \times 0.16} \times 2^{4 \times 0.18} = 2^{1.28} \times 2^{0.72} = 2^2 = 4$ .

4. Find the sum of the digits of the number  $4^{1011} \times 5^{2025}$ .

$4^{1011} \times 5^{2025}$  সংখ্যাটোৰ অঙ্কবোৰৰ যোগফল নিৰ্ণয় কৰা।

Ans :  $4^{1011} \times 5^{2025} = (2^2)^{1011} \times 5^{2025} = 2^{2022} \times 5^{2022} \times 5^3 = 10^{2022} \times 125$  which is 125 followed by 2022 zeros. Thus, the sum of the digits is  $1 + 2 + 5 + 0 + 0 + \dots + 0 = 8$ .

5. Arrange in descending order :  $\frac{4}{7}, \frac{5}{11}, \frac{7}{13}$ .

অধঃক্ৰমত (ডাঙৰৰ পৰা সৰু ক্ৰম অনুসৰি) সজোৱা।

Ans : Equating the denominator to the LCM of 7, 11, 13 i.e.  $1001 = 7 \times 11 \times 13$ , we get

$$\begin{aligned}\frac{4}{7} &= \frac{4 \times 11 \times 13}{7 \times 11 \times 13} = \frac{572}{1001} \\ \frac{5}{11} &= \frac{5 \times 7 \times 13}{11 \times 7 \times 13} = \frac{455}{1001} \\ \frac{7}{13} &= \frac{7 \times 7 \times 11}{13 \times 7 \times 11} = \frac{539}{1001}\end{aligned}$$

Thus,  $\frac{572}{1001} > \frac{539}{1001} > \frac{455}{1001} < \text{i.e. } \frac{4}{7} > \frac{7}{13} > \frac{5}{11}$ .

6. Let  $N_1, N_2, \dots, N_{100}$  be a sequence of 100 integers such that  $N_i + N_{i+1} = 2025$  for each  $i = 1, 2, \dots, 99$  i.e.  $N_1 + N_2 = 2025, N_2 + N_3 = 2025$  and so on. If  $N_{72} = 1$ , find  $N_{27}$ .

ধৰা হ'ল  $N_1, N_2, \dots, N_{100}$  এশটা অখণ্ড সংখ্যাৰ শ্ৰেণী, যাতে  $N_i + N_{i+1} = 2025$ , য'ত  $i = 1, 2, \dots, 99$  অৰ্থাৎ  $N_1 + N_2 = 2025, N_2 + N_3 = 2025$  ..ইত্যাদি। যদি  $N_{72} = 1$  হয় তেন্তে  $N_{27}$  ৰ মান নিৰ্ণয় কৰা।

Ans : Observe that  $N_1 + N_2 = 2025$  and  $N_2 + N_3 = 2025$ . Subtracting,  $(N_1 + N_2) - (N_2 + N_3) = 2025 - 2025 = 0$  so that  $N_1 - N_3 = 0$  which gives  $N_1 = N_3$ . Similarly,  $N_2 + N_3 = N_3 + N_4 = 2025$  which gives  $N_2 = N_4$ . Again,  $N_3 + N_4 = N_4 + N_5$  gives  $N_3 = N_5$  and  $N_4 + N_5 = N_5 + N_6$  gives  $N_4 = N_6$ . Proceeding this way, we can show  $N_1 = N_3 = N_5 = \dots = N_{100}$  i.e. all the odd numbered terms are equal and  $N_2 = N_4 = N_6 = \dots = N_{2024}$  i.e. all the even numbered terms are equal. Since  $N_{72} = 1$ , so  $N_{72} + N_{73} = 2025$  gives  $N_{73} = 2024$  and so all the odd numbered terms are equal to 2024. Hence,  $N_{27} = 2024$ .

7. Papon would be 5 minutes late to reach his destination if he rides his bike at 30 km per hour but he would be 10 minutes early if he rides at the speed of 40 km per hour. What is the distance of his destination from the starting point ?

যদি পাপনে বাইক এখন প্রতিঘণ্টাত 30 কি.মি. গতিবেগত চলায়, তেন্তে নিজৰ গন্তব্যস্থানত নিৰ্দিষ্ট সময়তকৈ 5 মিনিট পলমকৈ গৈ পাব আৰু প্রতি ঘণ্টাত 40 কি.মি. গতিবেগত চলালে 10 মিনিট আগতেই গৈ পালেহেতেন। তেওঁ যাত্ৰা আৰম্ভ কৰা ঠাইৰ পৰা গন্তব্যস্থানলৈ দূৰত্ব কিমান হ'ব নিৰ্ণয় কৰা।

Ans : Let the correct time to reach the destination be  $t$  hours i.e.  $60t$  minutes. Then,  $30(60t + 5) = 40(60t - 10)$ . Solving,  $1800t + 150 = 2400t - 400$  i.e.  $600t = 550$  i.e.  $t = \frac{55}{60}$  hours, which is equal to 55 minutes. Thus, Papon covers the required distance in  $55 + 5 = 60$  minutes or 1 hour by travelling 30 km per hour. Thus, the required distance is  $30 \times 1 = 30$  km.

8. A student took five papers in an examination, where the full marks were the same for each paper. His marks in these papers were in the proportion of 6 : 7 : 8 : 9 : 10. He obtained a total of  $\frac{3}{5}$  of the full marks in the five papers combined. In how many papers did he get more than 50% marks ?

এগৰাকী পৰীক্ষাৰ্থীয়ে পাঁচটা বিষয়ত পৰীক্ষাত লাভ কৰা নম্বৰৰ অনুপাত 6 : 7 : 8 : 9 : 10। প্রতিখন প্রশ্নকাকতৰ মুঠ নম্বৰৰ সংখ্যা একে। যদি পৰীক্ষাৰ্থী গৰাকীয়ে পোৱা সৰ্বমুঠ নম্বৰ আটাকেইখন প্রশ্ন কাকতৰ সৰ্বমুঠ নম্বৰৰ  $\frac{3}{5}$  অংশ হয়, তেন্তে পৰীক্ষাৰ্থী গৰাকীয়ে কিমানখন প্রশ্ন কাকতত 50 শতাংশ ৰ বেছি নম্বৰ লাভ কৰিছিল ?

Ans : Let the full marks in each paper be 100. The student obtained  $\frac{3}{5} \times 500 = 300$ . Let the marks obtained in the respective papers be  $6k, 7k, 8k, 9k$  and  $10k$ . Then  $6k + 7k + 8k + 9k + 10k = 300$  which gives  $k = \frac{30}{4}$ . So, marks in the first paper is  $6k = 6 \times \frac{30}{4} = 45$  out of 100 i.e. 45%.

Marks in the second paper is  $7k = 7 \times \frac{30}{4} = 52.5\% > 50\%$ . The marks in the remaining three

are  $8k$ ,  $9k$  and  $10k$  which are all greater than  $7k$  and hence more than 50%. Thus, the student scored more than 50% in four papers.

9. In an election of school captain, there were only two candidates. On the day of election, 10% of the students were absent. The winner was supported by 47% of the total students in the school. If he got support of 30 students more than his opponent, what is the number of students in the school ?

বিদ্যালয় এখনত বিদ্যালয় কেপ্টেইন (School Captain) নিৰ্বাচনত দুগৰাকী শিক্ষার্থীয়ে প্ৰাৰ্থী হিচাপে অৱতীৰ্ণ হৈছিল। নিৰ্বাচনৰ দিনত বিদ্যালয়ৰ 10 শতাংশ ছাত্ৰ-ছাত্ৰী অনুপস্থিত আছিল আৰু বিজয়ী প্ৰাৰ্থী গৰাকীয়ে বিদ্যালয়ৰ মুঠ ছাত্ৰ-ছাত্ৰীৰ 47 শতাংশৰ সমৰ্থনত নিৰ্বাচিত হ'ল। যদি তেওঁ প্ৰতিপক্ষৰ প্ৰাৰ্থীগৰাকীতকৈ 30 গৰাকী ছাত্ৰ-ছাত্ৰীৰ সমৰ্থন বেছিকৈ লাভ পালে তেন্তে বিদ্যালয়খনৰ সৰ্বমুঠ ছাত্ৰ ছাত্ৰীৰ সংখ্যা কিমান আছিল নিৰ্ণয় কৰা।

Ans : Since 10% students were absent and the winner got support from 47% students, so the opponent got support from  $100\% - 10\% - 47\% = 43\%$  students. By question  $47\% - 43\% = 4\%$  of the students is equal to 30. So, the number of students is  $\frac{100 \times 30}{4} = 750$ .

10. Let  $x, y, z$  be non-zero real numbers with  $x \neq z$  and  $\frac{x}{z} = \frac{x^2 + y^2}{z^2 + y^2}$  prove that

$$x^2 + y^2 + z^2 = (x + z - y)(x + z + y).$$

যদি  $x, y, z$  তিনিটা অশূন্য বাস্তৱ সংখ্যা য'ত  $x \neq z$  আৰু  $\frac{x}{z} = \frac{x^2 + y^2}{z^2 + y^2}$  প্ৰমাণ কৰা যে,

$$x^2 + y^2 + z^2 = (x + z - y)(x + z + y)$$

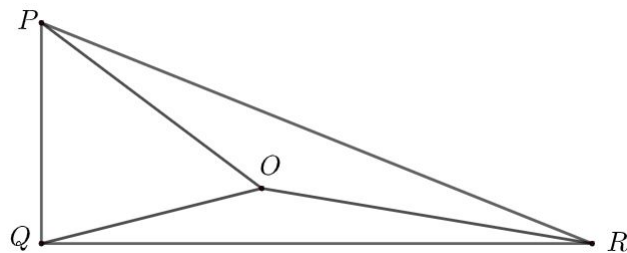
Ans :  $\frac{x}{z} = \frac{x^2 + y^2}{z^2 + y^2}$  gives  $xz^2 + xy^2 = x^2z + y^2z$  i.e.  $xz(z - x) = y^2(z - x)$  and as  $x \neq z$ , so  $xz = y^2$ .

Now,  $x^2 + y^2 + z^2 = x^2 + 2y^2 + z^2 - y^2 = x^2 + 2xz + z^2 - y^2 = (x + z)^2 - y^2 = (x + z - y)(x + z + y)$ .

11.  $\triangle PQR$  is right angled at  $Q$  with  $PR = 41$  and  $PQ = 9$ . If  $O$  is any point inside  $\triangle PQR$ , show that  $OP + OQ + OR > 45$ .

$Q$  বিন্দুত সমকোণ থকাকৈ  $PQR$  সমকোণী ত্ৰিভুজত  $PR = 41$  আৰু  $PQ = 9$ । যদি  $O$ ,  $PQR$  ত্ৰিভুজৰ অন্তৰ্ভূত এটি বিন্দু হয় দেখুওৱা যে,  $OP + OQ + OR > 45$ ।

Ans : By Pythagoras Theorem,  $PR^2 = PQ^2 + QR^2$  so that  $41^2 = 9^2 + QR^2$  i.e.  $QR^2 = 41^2 - 9^2 = (41 + 9)(41 - 9) = 50 \times 32 = 1600 = 40^2$ . So,  $QR = 40$ .

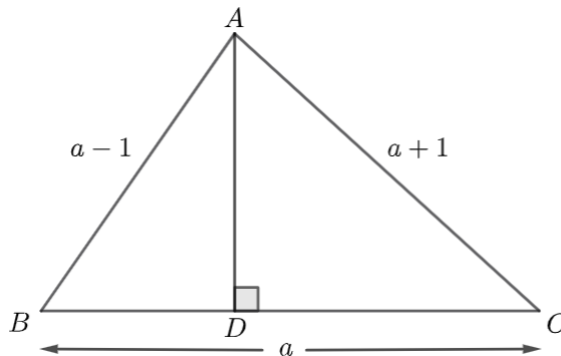


We now apply triangle inequality. In  $\triangle POQ$ ,  $OP + OQ > PQ$ . In  $\triangle QOR$ ,  $OQ + OR > QR$ . In  $\triangle ROP$ ,  $OR + OP > RP$ . Adding, we get  $2(OP + OQ + OR) > PQ + QR + RP$  i.e.  $2(OP + OQ + OR) > 9 + 40 + 41 = 90$ . Thus,  $OP + OQ + OR > 45$ .

12. The lengths of the sides of  $\triangle ABC$  are  $AB = a - 1$ ,  $BC = a$ ,  $CA = a + 1$  where  $a > 1$ .  $AD$  is drawn perpendicular to  $BC$ . Show that  $DC - BD = 4$ .

$ABC$  ত্ৰিভুজৰ বাহুবোৰৰ দীঘ এনেধৰণৰ,  $AB = a - 1$ ,  $BC = a$  আৰু  $CA = a + 1$ , য'ত  $a > 1$ ।  $BC$  ৰ ওপৰত  $AD$  লম্ব টনা হ'ল। দেখুওৱা যে,  $DC - BD = 4$ ।

Ans : In  $\triangle ABD$ , by Pythagoras Theorem,  $AD^2 = AB^2 - BD^2 = (a-1)^2 - BD^2$ . Again, in  $\triangle ACD$ ,  $AD^2 = AC^2 - DC^2 = (a+1)^2 - DC^2$ . Thus,  $(a-1)^2 - BD^2 = (a+1)^2 - DC^2$  which gives  $DC^2 - BD^2 = (a+1)^2 - (a-1)^2$  so that  $(DC+BD)(DC-BD) = (a+1+a-1)(a+1-a-1)$  i.e.  $a(DC - BD) = 4a$ . So,  $DC - BD = 4$ .



13. The present age of a father is three times that of his son. After 10 years, the ratio of their ages will be 13 : 6. Find their present ages.

এগৰাকী ব্যক্তিৰ বৰ্তমান বয়স তেওঁৰ পুত্ৰৰ বয়সৰ তিনিগুণ। 10 বছৰ পিছত পিতা-পুত্ৰৰ বয়সৰ অনুপাত 13 : 6 হ'লে তেওঁলোকৰ বৰ্তমান বয়স কিমান ?

Ans : Let the present age of the son be  $x$ . So, present age of the father is  $3x$ . By question,  $\frac{3x+10}{x+10} = \frac{13}{6}$ . This gives,  $6(3x+10) = 13(x+10)$  i.e.  $18x+60 = 13x+130$  i.e.  $5x = 70$  so that  $x = 14$ . Thus, son's age is 14 and father's age is  $3 \times 14 = 42$ .

14. If  $a^2 + b^2 = \frac{1013ab}{506}$  where  $a > b > 0$ , find the value of  $\frac{a+b}{a-b}$ .

যদি  $a^2 + b^2 = \frac{1013ab}{506}$  হয়, য'ত  $a > b > 0$ , তেন্তে  $\frac{a+b}{a-b}$ -ৰ মান উলিওৱা।

Ans : Observe that  $a^2 + b^2 - 2ab = \frac{1013ab}{506} - 2ab = \frac{ab}{506}$ . Also,  $a^2 + b^2 + 2ab = \frac{1013ab}{506} + 2ab = \frac{2025ab}{506}$ . Thus,  $\frac{(a+b)^2}{(a-b)^2} = 2025 = 45^2$ . Since  $a > b > 0$ , so  $\frac{a+b}{a-b} = 45$ .

15. Find all integers  $n$  for which  $(n^2 - 3n + 1)^2 + 1$  is a prime number.

$n$  ৰ আটাইবোৰ অখণ্ড মান উলিওৱা যাতে  $(n^2 - 3n + 1)^2 + 1$  এটি মৌলিক সংখ্যা হয়।

Ans : Observe that  $(n^2 - 3n + 1)^2 + 1 = \{n(n-3) + 1\}^2 + 1$ . If  $n$  is even, then  $n(n-3)$  is even. If  $n$  is odd then  $n-3$  is even and hence  $n(n-3)$  is even. Thus,  $n(n-3)$  is always even so that  $n(n-3) + 1$  is always odd. Hence,  $\{n(n-3) + 1\}^2 + 1$  is always even. So, it can be prime if and only if  $(n^2 - 3n + 1)^2 + 1 = 2$ . This gives  $(n^2 - 3n + 1)^2 = 1$ , so that  $n^2 - 3n + 1 = 1$  or  $n^2 - 3n + 1 = -1$ . If  $n^2 - 3n + 1 = 1$  then  $n^2 - 3n = 0$  i.e.  $n(n-3) = 0$  so that  $n = 0$  or  $n = 3$ . If  $n^2 - 3n + 1 = -1$  then  $n^2 - 3n + 2 = 0$  i.e.  $(n-1)(n-2) = 0$  so that  $n = 1$  or  $n = 2$ . Thus, the required integers for which  $(n^2 - 3n + 1)^2 + 1$  is a prime number are  $n = 0, 1, 2, 3$ .

16. Find the number of ways of distributing 12 identical (equal in all respect) balls among 4 boys so that every boy gets at least two balls but no one gets more than 4.

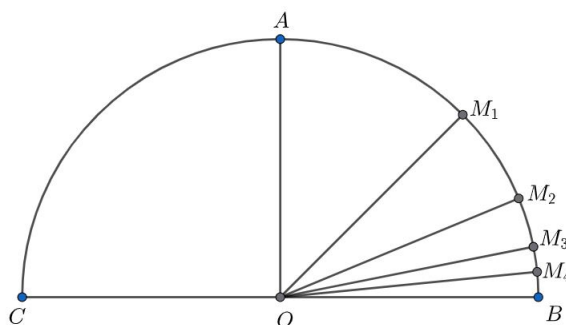
4 জন ল'ৰাৰ মাজত 12 টা অবিভল একে বল (সকলো পিনৰ পৰা সমান) কিমান ধৰণে বিতৰণ কৰিব পাৰি উলিওৱা যাতে প্ৰতিজন ল'ৰাই কমপক্ষে দুটা বল পায়, কিন্তু কৰে কোনেও 4 টাতকৈ অধিক নাপায়।

Ans : If the first boy gets 2 balls and the second boy gets 2 balls, then the remaining 8 balls has to be distributed 4 each to the third boy and the fourth boy as  $2 + 2 + 4 + 4 = 12$ .

We denote it as  $(2, 2, 4, 4)$ . The other distributions can be  $(2, 4, 2, 4)$ ,  $(2, 4, 4, 2)$ ,  $(4, 2, 4, 2)$ ,  $(4, 2, 2, 4)$  and  $(4, 4, 2, 2)$ . Thus, there are 6 ways where 2 balls are given to two boys and 4 balls to the remaining two. Those who are comfortable with permutations with repetitions can compute the number of arrangements of  $(2, 2, 4, 4)$  as  $\frac{4!}{2!2!} = 6$ . Next, we can have the distribution  $(2, 3, 3, 4)$  which can be of  $\frac{4!}{2!} = 12$  ways i.e.  $(2, 3, 3, 4)$ ,  $(2, 3, 4, 3)$ ,  $(2, 4, 3, 3)$ ,  $(3, 2, 3, 4)$ ,  $(3, 2, 4, 3)$ ,  $(3, 3, 4, 2)$ ,  $(3, 3, 2, 4)$ ,  $(3, 4, 2, 3)$ ,  $(3, 4, 3, 2)$ ,  $(4, 2, 3, 3)$ ,  $(4, 3, 2, 3)$  and  $(4, 3, 3, 2)$ . Next, we can have  $(3, 3, 3, 3)$  which can be of 1 way only. Hence, the total number of ways of distributing 12 identical balls among 4 boys so that every boy gets at least two balls but no one gets more than 4 is  $6 + 12 + 1 = 19$ .

17.  $A$  is the midpoint of the semi-circular arc with centre  $O$  and diameter  $BC$ .  $M_1$  is the midpoint of the arc  $AB$ ,  $M_2$  is the midpoint of the arc  $M_1B$ ,  $M_3$  is the midpoint of the arc  $M_2B$  and  $M_4$  is the midpoint of the arc  $M_3B$ . Find  $\angle AOM_1, \angle AOM_2, \angle AOM_3, \angle AOM_4$  as fractional multiples of  $90^\circ$ . An example of fractional multiple of  $90^\circ$  is  $45^\circ = \frac{1}{2} \times 90^\circ$ .

$O$  কেন্দ্ৰ আৰু  $BC$  ব্যাসযুক্ত অৰ্ধবৃত্তাকাৰ চাপ এটাৰ  $A$  মধ্যবিন্দু। চাপ  $AB$  ৰ মধ্যবিন্দু হ'ল  $M_1$ , চাপ  $M_1B$  ৰ মধ্যবিন্দু  $M_2$ , চাপ  $M_2B$  ৰ মধ্যবিন্দু  $M_3$  আৰু চাপ  $M_3B$  ৰ মধ্যবিন্দু  $M_4$ ।  $\angle AOM_1, \angle AOM_2, \angle AOM_3, \angle AOM_4$  কোণসমূহৰ মান  $90^\circ$  ৰ ভগ্নাংশ গুণিতক হিচাপে লিখা।  $90^\circ$  ৰ ভগ্নাংশ গুণিতকৰ এটি উদাহৰণ যেনে  $45^\circ = \frac{1}{2} \times 90^\circ$ .



Ans : We have  $\angle AOB = 90^\circ$ .  $OM_1$  bisects  $\angle AOB$ . So,  $\angle AOM_1 = \angle M_1OB = \frac{1}{2} \times 90^\circ$ . Next,  $OM_2$  bisects  $\angle M_1OB$ . So,  $\angle M_1OM_2 = \angle M_2OB = \frac{1}{2} \times \frac{1}{2} \times 90^\circ = \frac{1}{2^2} \times 90^\circ$ . Thus,  $\angle AOM_2 = \angle AOM_1 + \angle M_1OM_2 = \left( \frac{1}{2} \times 90^\circ + \frac{1}{2^2} \times 90^\circ \right) = \frac{3}{2^2} \times 90^\circ$ . Next,  $\angle M_2OM_3 = \frac{1}{2} \times \angle M_2OB = \frac{1}{2} \times \frac{1}{2^2} \times 90^\circ = \frac{1}{2^3} \times 90^\circ$ . So,  $\angle AOM_3 = \angle AOM_2 + \angle M_2OM_3 = \left( \frac{3}{2^2} \times 90^\circ + \frac{1}{2^3} \times 90^\circ \right) = \frac{7}{2^3} \times 90^\circ$ . Similarly,  $\angle AOM_4 = \angle AOM_3 + \angle M_3OM_4 = \left( \frac{7}{2^3} \times 90^\circ + \frac{1}{2^4} \times 90^\circ \right) = \frac{15}{2^4} \times 90^\circ$ . Observe that  $\angle AOM_1 = \frac{2-1}{2} \times 90^\circ$ ,  $\angle AOM_2 = \frac{2^2-1}{2^2} \times 90^\circ$ ,  $\angle AOM_3 = \frac{2^3-1}{2^3} \times 90^\circ$ ,  $\angle AOM_4 = \frac{2^4-1}{2^4} \times 90^\circ$ .

(Additional remark : Observe that  $\angle AOM_n = \frac{2^n - 1}{2^n} \times 90^\circ$ . To justify the general expression above, we use the principle of mathematical induction. Assume that  $\angle AOM_k = \frac{2^k - 1}{2^k} \times 90^\circ$  for some natural number  $k$ . This assumption is fine as we have already tested it for  $k = 1, 2, 3, 4$ . Then,

$\angle AOM_{k+1} = \angle AOM_k + \angle M_k OM_{k+1} = \left( \frac{2^k - 1}{2^k} \times 90^\circ + \frac{1}{2} \times \frac{1}{2^k} \times 90^\circ \right) = \frac{2^{k+1} - 1}{2^{k+1}}$ . Hence, the expression for  $\angle AOM_{k+1}$  is true whenever the expression for  $\angle AOM_k$  is true. Thus, the Principle of Mathematical Induction gives  $\angle AOM_n = \frac{2^n - 1}{2^n} \times 90^\circ$  for every natural number  $n$ .)

18. There are 10000 i.e.  $100^2$  birds sitting in a row. A gun shot is fired and because of this the birds sitting in perfect square positions i.e. the 1st, 4th, 9th, 16th, ...,  $100^2$ th fly away. The remaining birds again rearrange themselves in a row. A second gun shot is fired and again the birds in the perfect square positions fly away. This process is repeated until the number of remaining birds drops below 100. How many birds remain as soon as the number of birds comes below 100 ?

এটা শাৰীত 10000 টা অৰ্থাৎ  $100^2$  টা চৰাই বহি আছিল। এটা বন্দুকৰ গুলীৰ শব্দত পূৰ্ণ বৰ্গ স্থানত বহি থকা (যেনে প্রথম, চতুৰ্থ, নৱম, 16 সংখ্যক....) চৰাইবোৰ উৰি গ'ল। বাকী চৰাইবোৰ আকৌ এটা শাৰীত নতুন সজ্জাত বহিল। দ্বিতীয় বাৰ গুলিৰ শব্দত আকৌ পূৰ্ণ বৰ্গ স্থানত বহি থকা চৰাইবোৰ উৰি গ'ল। এনেদৰে চৰাইৰ সংখ্যা 100 ৰ তলত নামি নহালৈকে এই প্ৰক্ৰিয়া অব্যাহত থাকিল। প্ৰথমবাৰৰ বাবে চৰাইৰ সংখ্যা 100 ৰ তলত নামি অহাৰ সময়ত চৰাইৰ সংখ্যা কিমান আছিল ?

Ans : There are exactly 100 perfect squares from 1 to  $100^2$ . So, the number of birds remaining after the first shot is  $100^2 - 100 = 100 \times 99$ . Now there are exactly 99 perfect squares from 1 to  $100 \times 99$ . So the number of birds remaining after the second shot is  $100 \times 99 - 99 = 99^2$ . Next, there are exactly 99 perfect squares from 1 to  $99^2$ . So, the number of birds remaining after the third shot is  $99^2 - 99 = 99 \times 98$ . After the fourth shot, the number of birds remaining is  $99 \times 98 - 98 = 98^2$ . We can analyse this by starting with  $n^2$  birds. Birds remaining after first shot is  $n^2 - n = n(n - 1)$ . Birds remaining after second shot is  $n(n - 1) - (n - 1) = (n - 1)^2$ . After the third shot,  $(n - 1)^2 - (n - 1) = (n - 1)(n - 2)$  birds remain. After the fourth shot,  $(n - 2)^2$  birds remain. As this process continues, we can write the number of birds remaining after the subsequent shots as

$$100^2 \rightarrow 100 \times 99 \rightarrow 99^2 \rightarrow 99 \times 98 \rightarrow 98^2 \rightarrow 98 \times 97 \rightarrow 97^2 \rightarrow \dots \rightarrow 10^2 \rightarrow 10 \times 9 \rightarrow \dots$$

Thus, the number of birds that remain as soon as the number of birds comes below 100 is  $10 \times 9 = 90$ .