Assam Academy of Mathematics
Assam Mathematics Olympiad 2025
Category III (Classes IX - XI)
24th August 2025

Full marks : 100 Time : 3 hours

There are 18 questions. Questions 1 to 5 carry 2 marks each. Questions 6 to 13 carry 5 marks each. Questions 14 to 18 carry 10 marks each.

ইয়াত 18 টা প্ৰশ্ন আছে। 1 ৰ পৰা 5 লৈ প্ৰতিটো প্ৰশ্নত 2 নম্বৰকৈ আছে। 6 ৰ পৰা 13 লৈ প্ৰতিটো প্ৰশ্নত 5 নম্বৰকৈ আছে। আৰু 14 ৰ পৰা 18 লৈ প্ৰতিটো প্ৰশ্নত 10 নম্বৰকৈ আছে।

There may be various other ways of solutions than those shown here. Queries or suggestions regarding the solutions can be mailed to mail@aamonline.in
ইয়াত দেখুওৱা ধৰণবিলাকৰ বাহিৰেও প্ৰশ্নবোৰৰ সমাধানৰ আন বিভিন্ন উপায় থাকিব পাৰে। সমাধানবোৰৰ বিষয়ে কিবা প্ৰশ্ন বা পৰামৰ্শ থাকিলে mail@aamonline.in লৈ মেইল কৰিব পাৰে।

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1. If 25 and 81 are the roots of the equation $x^2+bx+c=0$, find the values of b and c.

   25 আৰু 81, $x^2+bx+c=0$ সমীকৰণটোৰ মূল হ'লে b আৰু c ৰ মান উলিওৱা ।

   Ans : Given equation is $x^2+bx+c=0$. Its roots are 25 and 81. Then, sum of roots is $-\frac{b}{1}=25+81=106$ so that $b=-106$. Also, product of roots is $\frac{c}{1}=25\times 81=2025$, so that $c=2025$.

2. Find all the solutions of the equation : $|x-1|=2x-1$.

   $|x-1|=2x-1$ সমীকৰণটোৰ আটাইবোৰ মূল নিৰ্ণয় কৰা ।

   Ans : $|x-1|=2x-1$ implies either $2x-1=x-1$ or $2x-1=-(x-1)$. If $2x-1=x-1$ then $x=0$ but we see that $x=0$ is not a solution of the original equation as $|0-1|=1\neq 2\times 0-1=-1$. If $2x-1=-(x-1)$, then $3x=2$ i.e. $x=\frac{2}{3}$ which satisfies the given equation.

3. If a,b,c are positive real numbers, find the minimum value of
   $$\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc}.$$
   a,b,c যিকোনো ধনাত্মক বাস্তৱ সংখ্যাৰ বাবে
   $$\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc}$$
   ৰ সৰ্বনিম্ন মান উলিওৱা ।

   Ans : Using A.M-G.M inequality, $a+\frac{1}{a}\geq 2\times\sqrt{a\times\frac{1}{a}}=2$, so that $a+\frac{1}{a}+3\geq 5$. Similarly, $b+\frac{1}{b}+3\geq 5$ and $c+\frac{1}{c}+3\geq 5$. Multiplying, we get
   $\left(a+\frac{1}{a}+3\right)\left(b+\frac{1}{b}+3\right)\left(c+\frac{1}{c}+3\right)\geq 125$ which gives
   $\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc}\geq 125$. Also, we see that for $a=b=c=1$, the value of the expression is 125. Hence, the required minimum value is 125. It is to be noted here that only the argument that the expression is greater than or equal to 125 is not sufficient to claim that the minimum value is 125. That the value is indeed attained gives us the guarantee

that the minimum value is 125. For example, applying the AM-GM inequality to $a, \frac{1}{a}, 3$, we get $\left(a+\frac{1}{a}+3\right) \geq 3\left(a \cdot \frac{1}{a} \cdot 3\right)^{1/3}$. Applying the same for b and c terms, we get

$\left(a+\frac{1}{a}+3\right)\left(b+\frac{1}{b}+3\right)\left(c+\frac{1}{c}+3\right) \geq 27 \times (27)^{1/3} = 81.$

But the minimum value is not 81 as the expression is always greater than or equal to 125.

4. Find the number of integers x such that $2^{2x} - 3 \times 2^{x+2} + 32 = 0$.
   x ৰ অখণ্ড সাংখ্যিক মানৰ বাবে $2^{2x} - 3 \times 2^{x+2} + 32 = 0$ সমীকৰণটোৰ সমাধান উলিওৱা ।

   Ans : We have: $2^{2x} - 3(2^{x+2}) + 32 = 0$. Let $y = 2^x$ so that $y^2 - 3(4y) + 32 = 0$ i.e. $y^2 - 12y + 32 = 0$ i.e. $(y-8)(y-4) = 0$ i.e $y = 8, 4$. Thus, $2^x = 8$ or $2^x = 4$ i.e $x = 3, 2$. So, there are only 2 such integers.

5. Let $d_1, d_2, \ldots, d_k$ be all the positive factors of a positive integer n, including 1 and n. Suppose $d_1 + d_2 + \cdots + d_k = 144$, find the value of $\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_k}$.
   ধৰা হ'ল, এটা ধনাত্মক অখণ্ড সংখ্যা n ৰ আটাইবোৰ ধনাত্মক উৎপাদক (1 আৰু n কে ধৰি) $d_1, d_2, \ldots, d_k$ । যদি $d_1 + d_2 + \cdots + d_k = 144$ তেন্তে $\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_k}$ ৰ মান উলিওৱা ।

   Ans : We have $d_1 + d_2 + d_3 + \ldots + d_k = 144$. Observe that for each d_i, the number $\frac{n}{d_i}$ is also a positive factor of n as $d_i \times \frac{n}{d_i} = n$. For each d_i, the corresponding $\frac{n}{d_i}$ is unique. Thus, all possible factors of n can also be listed as $\frac{n}{d_1}, \frac{n}{d_2}, \ldots, \frac{n}{d_k}$, so that $\frac{n}{d_1} + \frac{n}{d_2} + \frac{n}{d_3} + \ldots + \frac{n}{d_k} = 144$ which gives $\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_k} = \frac{144}{n}$.

6. A club with N members is organized into four sub-committees according to the following rules:
   (a) Each member belongs to exactly two sub-committees.
   (b) Each pair of sub-committees has exactly one member in common.

   Then find the value of N.
   N সদস্যযুক্ত এটি সংঘৰ সদস্যসকলক লৈ তলত উল্লেখিত বিধিৰ আধাৰত চাৰিখন উপসমিতি গঠন কৰা হ'ল:
   (a) প্ৰতিগৰাকী সদস্য সঠিকভাৱে দুটা উপসমিতিৰ অন্তৰ্ভুক্ত ।
   (b) প্ৰতিযোৰ উপসমিতিত সঠিকভাৱে এগৰাকী সাধাৰণ সদস্য অন্তৰ্ভুক্ত ।

   N ৰ মান উলিওৱা ।

   Ans : Let C_i = Set of members in the ith committee, $i = 1, 2, 3, 4$. So, $|C_1 \cup C_2 \cup C_3 \cup C_4| = |\cup_{i=1}^{n} C_i| = N$. Since each member belongs to exactly two sub-committees, so each member is counted twice in the sum $\sum_{i=1}^{4}|C_i|$, so that $\sum_{i=1}^{4}|C_i| = 2N$. Since each pair of sub-committees has exactly one member in common, so $|C_i \cap C_j| = 1$ for $i \neq j$, $|C_i \cap C_j \cap C_k| = 0$ and $|C_1 \cap C_2 \cap C_3 \cap C_4| = 0$. Using inclusion-exclusion principle,

$$|C_1 \cup C_2 \cup C_3 \cup C_4| = \sum_{i=1}^{4}|C_i| - \sum_{i,j=1}^{4}|C_i \cap C_j| + \sum_{i,j,k=1}^{4}|C_i \cap C_j \cap C_k| - |C_1 \cap C_2 \cap C_3 \cap C_4|$$

$\Rightarrow N = 2N - {}^4C_2 \times 1 + 0 - 0 \Rightarrow N = 6.$

7. Compute the sum $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2$. (Hint : $n^2 - (n+1)^2 = -2n - 1$).

যোগফল নিৰ্ণয় কৰা : $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2$ । (ইঙ্গিত : $n^2 - (n+1)^2 = -2n - 1$).

Ans : Let $S = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \cdots + 99^2$. We can group the terms in pairs, such as: $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \cdots + (97^2 - 98^2) + (99^2)$. Note that, $n^2 - (n+1)^2 = -2n - 1$. We evaluate each pair:

$$1^2 - 2^2 = -2 \times 1 - 1$$

$$3^2 - 4^2 = -2 \times 3 - 1$$

$$5^2 - 6^2 = -2 \times 5 - 1$$

$$\cdots\cdots\cdots$$

$$97^2 - 98^2 = -2 \times 97 - 1$$

Since $97 = 2 \times 49 - 1$, so there are 49 expressions above. Thus,

$(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \cdots + (97^2 - 98^2) = -2(1 + 3 + 5 + \ldots + 97) - 49$

Now, $1+3+5+\ldots+97 = 49^2$. So, $S = -2 \times 49^2 - 49 + 99^2 = -49(98+1) + 99^2 = 99(99-49) = 99 \times 50 = 4950$.

8. If p and q are prime numbers such that $p^2 + pq + q^2$ is the square of a positive integer, then find the value of $p^2 - pq + q^2$.

যদি p আৰু q এনেকুৱা দুটা মৌলিক সংখ্যা যাতে $p^2 + pq + q^2$ এটি ধনাত্মক অখণ্ড সংখ্যাৰ বৰ্গফলৰ সমান হয় তেন্তে $p^2 - pq + q^2$ ৰ মান উলিওৱা ।

Ans : Let $p^2 + pq + q^2 = k^2$ where k is a positive integer. Then, $p^2 + 2pq + q^2 - pq - k^2 = 0$ i.e. $(p+q)^2 - k^2 = pq$ i.e. $(p+q+k)(p+q-k) = pq$. But $p+q+k > p$ and $p+q+k > q$. Also, p and q are primes and prime factorization of a number is unique. So, the equality implies that $p+q+k = pq$ and $p+q-k = 1$. Adding, $2(p+q) = 1 + pq$ i.e. $pq - 2p - 2q + 4 - 3 = 0$ i.e. $(p-2)(q-2) = 3$ but 3 is prime. So, either $p - 2 = 1, q - 2 = 3$ or $p - 2 = 3, q - 2 = 1$ i.e. $p = 3, q = 5$ or $p = 5, q = 3$. In both cases, $p^2 - pq + q^2 = 3^2 + 5^2 - 3 \times 5 = 19$.

9. Show that $x^3 + y^3 + z^3 = 1111$ has no integer solutions.

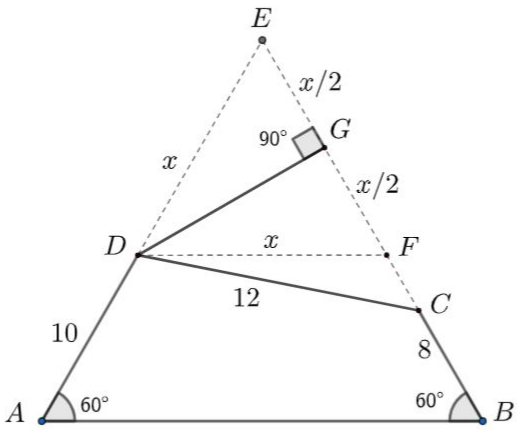
দেখুওৱা যে $x^3 + y^3 + z^3 = 1111$ ৰ কোনো অখণ্ড সমাধান নাই।

Ans : Cube of any integer is of the form $9k$ or $9k+1$ or $9k-1$. In terms of congruence, $x^3 \equiv 0$ or 1 or $-1 \pmod 9$, $y^3 \equiv 0$ or 1 or $-1 \pmod 9$ and $z^3 \equiv 0$ or 1 or $-1 \pmod 9$. It can be checked that $x^3 + y^3 + z^3 \equiv -3$ or -2 or -1 or 0 or 1 or 2 or $3 \pmod 9$. But $1111 \equiv 4 \pmod 9$. So, the equality $x^3 + y^3 + z^3 = 1111$ cannot hold for integers.

10. In quadrilateral $ABCD$, $BC = 8, CD = 12, AD = 10$, $\angle A = \angle B = 60^\circ$. Find AB. Hint : Extend AD and BC to meet at E.

$ABCD$ চতুৰ্ভুজৰ $BC = 8, CD = 12, AD = 10, \angle A = \angle B = 60^\circ$ । AB উলিওৱা । (ইঙ্গিত : AD আৰু BC ক E বিন্দুত লগলগাকৈ বঢ়াই দিয়া ।)

Ans : Observe that if we extend AD and BC to meet at E, then $\triangle ABE$ is equilateral. We construct DF parallel to AB. Then, $\triangle DFE$ is also equilateral. Let $DE = EF = DF = x$. We construct DG perpendicular to EF.



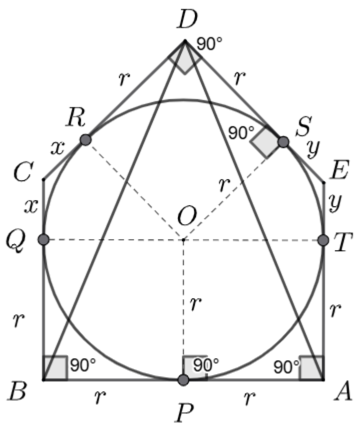
It can be observed that since DF is parallel to AB and $\triangle ABE$ is equilateral, so $AD = BF = 10$ so that $FC = BF - BC = 10 - 8 = 2$. Since $\triangle DFE$ is equilateral, so DG bisects EF. So, $EG = FG = \dfrac{x}{2}$. In $\triangle DGF$, by Pythagoras theorem, $DG^2 = DF^2 - FG^2$, so that $DG^2 = x^2 - \left(\dfrac{x}{2}\right)^2 = \dfrac{3x^2}{4}$. (This is actually a standard result regarding altitude of an equilateral triangle. Students who know it can use directly.)

Next, in $\triangle DGC$, by Pythagoras theorem, $DG^2 + GC^2 = DC^2$ i.e. $\dfrac{3x^2}{4} + (GF + FC)^2 = 12^2$ i.e. $\dfrac{3x^2}{4} + \left(\dfrac{x}{2} + 2\right)^2 = 144$. This gives $\dfrac{3x^2}{4} + \dfrac{x^2}{4} + 2 \times \dfrac{x}{2} \times 2 + 2^2 = 144$ i.e. $x^2 + 2x = 140$. Adding 1 to both sides, we can get $x^2 + 2x + 1 = 141$ which gives $(x+1)^2 = 141$ so that $x = \sqrt{141} - 1$. Thus, $AB = AE = AD + DE = 10 + x = 10 + \sqrt{141} - 1 = 9 + \sqrt{141}$.

Alternatively, students familiar with the cosine formula of triangles can do it very simply by considering $\triangle DEC$ as $DE^2 + CE^2 - DC^2 = 2 \times DE \times CE \times \cos\angle DEC$ which gives $x^2 + (x+2)^2 - 12^2 = 2x(x+2)\cos 60^\circ$ i.e. $x^2 + x^2 + 4x + 4 - 144 = 2x(x+2) \times \dfrac{1}{2}$ i.e. $x^2 + 2x = 140$ i.e. $x^2 + 2x + 1 = 141$ and so $x = \sqrt{141} - 1$ which gives $AB = 9 + \sqrt{141}$.

11. A circle is inscribed inside a pentagon $ABCDE$ where $\angle ABC = \angle CDE = \angle EAB = 90^\circ$. Find $\angle ADB$.

এটি অন্তৰ্লিখিত বৃত্ত আঁকিব পৰাকৈ এটি পঞ্চভুজ $ABCDE$ অঁকা হ'ল যাতে $\angle ABC = \angle CDE = \angle EAB = 90^\circ$ । $\angle ADB$ নিৰ্ণয় কৰা ।



Ans : Let O be the centre of the circle. Since each side of the pentagon is a tangent to the inscribed circle, we mark the points of contact of AB, BC, CD, DE, EA as P, Q, R, S, T

respectively. Since radius is perpendicular to the tangent at the point of contact, so it follows that $OPAT, OQBP$ and $ORDS$ are squares. Also, tangents from an external point to a circle are of equal length. So, we can mark the lengths of the segments of the sides of the pentagon made by the points of contact with quantities x, y and r as shown in the figure. It follows that $\triangle BCD$ and $\triangle DEA$ are isosceles. So, $\angle BDC = 90^\circ - \frac{\angle BCD}{2}$ and $\angle ADE = 90^\circ - \frac{\angle AED}{2}$. Thus, $\angle ADB = 90^\circ - \angle BDC - \angle ADE = \frac{\angle BCD}{2} + \frac{\angle AED}{2} - 90^\circ$. But in pentagon $ABCDE$, angle sum property gives $\angle ABC + \angle BCD + \angle CDE + \angle DEA + \angle EAB = 540^\circ$ so that $\angle BCD + \angle DEA = 270^\circ$. Hence $\angle ADB = \frac{270^\circ}{2} - 90^\circ = 45^\circ$.

12. In a multiple-choice test there are 8 questions. Each question has 4 options, of which only one option is correct. If a candidate answers all the questions by choosing one option for each, then find the number of ways of choosing the options so that exactly 4 questions are answered correctly.

বহু বাছনিযুক্ত এখন প্ৰশ্নকাকতত 8 টা প্ৰশ্ন আছে। প্ৰতিটো প্ৰশ্নৰে চাৰিটাকৈ উত্তৰ দিয়া আছে যাৰ কেৱল এটাহে শুদ্ধ। যদি এজন প্ৰাৰ্থীয়ে প্ৰতিটো প্ৰশ্নৰে এটাকৈ উত্তৰ বাছনি কৰি আটাইবোৰ প্ৰশ্নৰ উত্তৰ লিখে তেন্তে তেওঁ মুঠ কিমান ধৰণে উত্তৰসমূহ বাছিব পাৰিব যাতে তেওঁৰ সঠিকভাৱে 4 টা প্ৰশ্নৰ উত্তৰ শুদ্ধ হয় ।

Ans : The correct option for each question can be chosen in 1 way only. Out of 8 questions, 4 questions that are to be answered correctly can be chosen in ${}^8C_4 = \dfrac{8!}{4!4!} = \dfrac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70$ ways. Now, out of 4 options in a question, there are 3 incorrect options. For each combination of 4 questions answered correctly, the number of ways of choosing incorrect options for the remaining 4 questions is $3 \times 3 \times 3 \times 3 = 81$. Hence, the number of ways of choosing the options so that exactly 4 questions are answered correctly is $70 \times 81 = 5670$.

13. Find the greatest two-digit number n such that the square of the sum of the digits of n is equal to the sum of digits of n^2. Justify your answer.

দুই অংকীয়া বৃহত্তম সংখ্যা n উলিওৱা যাতে n ৰ অংক দুটাৰ যোগফলৰ বৰ্গ n^2 ৰ অংককেইটাৰ সমষ্টিৰ সমান হয় । তোমাৰ উত্তৰৰ বৈধতা প্ৰতিপন্ন কৰা ।

Ans : The greatest two digit number is 99 and $99^2 = 9801$ which is of 4 digits. Thus, for any two digit number n, the number of digits of n^2 is at most 4. So, sum of the digits of n^2 is at most $9+9+9+9 = 36$. So, square of the sum of the digits of the required n is at most 36. So, the sum of the digits of n can be $1, 2, 3, 4$ or 5 only. It cannot be 6 as the maximum possible n^2 is 9801 which is less than 9999. So, all possible two digit numbers can be listed as $50, 41, 14, 32, 23, 40, 31, 13, 22, 30, 21, 12, 20, 11, 10$. We list them in decreasing order to check for the largest n for which the square of the sum of the digits of n is equal to the sum of digits of n^2. The list is $50, 41, 40, 32, 31, 30, 23, 22, 21, 20, 14, 13, 12, 11, 10$. We denote the sum of digits of n as $s(n)$. We have $s(50^2) = s(2500) = 7 \neq (s(50))^2 = 25$. $s(41^2) = s(1681) = 16 \neq (4+1)^2 = 25$. $s(40^2) = s(1600) = 7 \neq (4+0)^2 = 16$. $s(32^2) = s(1024) = 7 \neq (3+2)^2 = 25$. $s(31^2) = s(961) = 16 = (3+1)^2 = 16$. Thus, the largest such n is 31.

14. For each integer n, let $f(n) = \dfrac{1}{3^n + \sqrt{3}}$. Show that

প্ৰতিটো অখণ্ড সংখ্যা n ৰ বাবে, ধৰা হ'ল $f(n) = \dfrac{1}{3^n + \sqrt{3}}$ । দেখুওৱা যে

$\sqrt{3}\,[f(-5) + f(-4) + f(-3) + \ldots + f(4) + f(5) + f(6)] = 6.$

Ans : $f(n) = \dfrac{1}{3^n + \sqrt{3}}$. So, $f(1-n) = \dfrac{1}{3^{1-n} + \sqrt{3}} = \dfrac{3^n}{3 + 3^n\sqrt{3}} = \dfrac{3^n}{\sqrt{3}(3^n + \sqrt{3})}$. Thus, $f(n) + f(1-n) = \dfrac{1}{3^n + \sqrt{3}}\left(1 + \dfrac{3^n}{\sqrt{3}}\right) = \dfrac{1}{\sqrt{3}}$. Now,

$$
\begin{aligned}
&\sqrt{3}\,[f(-5)+f(-4)+f(-3)+\ldots+f(4)+f(5)+f(6)]\\
=&\sqrt{3}\,([f(6)+f(-5)]+[f(5)+f(-4)]+[f(4)+f(-3)]+\\
&[f(3)+f(-2)]+[f(2)+f(-1)]+[f(1)+f(0)])\\
=&\sqrt{3}\times 6\times\frac{1}{\sqrt{3}}=6
\end{aligned}
$$

15. Consider polynomials of the form $p(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ where $a_0 < a_1 < \ldots < a_n$ are all possible positive divisors of a_n and $a_n \leq 2025$ and further, two of the coefficients are 14 and 17. For example, $1+2x+7x^2+14x^3+17x^4+34x^5+119x^6+238x^7$ is one such polynomial whereas $1+2x+7x^2+14x^3+17x^4+34x^5$ is not as 7 is not a divisor of 34. What is the largest possible degree of such polynomials ?

$p(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ আকাৰৰ বহুপদ সমূহ বিবেচনা কৰা যাতে $a_0 < a_1 < \ldots < a_n$ আদি সহগবোৰৰ আটায়েই a_n ৰ সম্ভৱপৰ ধনাত্মক ভাজক আৰু $a_n \leq 2025$ । লগতে সহগসমূহৰ ভিতৰত দুটাৰ মান 14 আৰু 17 । উদাহৰণস্বৰূপে $1+2x+7x^2+14x^3+17x^4+34x^5+119x^6+238x^7$ এনে ধৰণৰ এটি বহুপদ। আনহাতে, $1+2x+7x^2+14x^3+17x^4+34x^5$ এনে ধৰণৰ এটি বহুপদ নহয় কিয়নো 7, 34 ৰ ভাজক নহয়। এনেকুৱা বহুপদ সমূহৰ সম্ভৱপৰ সৰ্বোচ্চ ঘাত কিমান ?

Ans : Since $14 = 2 \times 7$ and 17 are two of the coefficients, so a_n must have $2, 7$ and 17 as three of its prime factors. The least such a_n is $2 \times 7 \times 17 = 238$. To find the number of divisors, one way is to list all the divisors by successive splitting as a product of two factors. $238 = 1 \times 238 = 2 \times 119 = 7 \times 34 = 14 \times 17$. The smarter way is to use the multiplication rule of counting. Any divisor of $2 \times 7 \times 17$ will be of the form $2^x \times 7^y \times 17^z$ where x takes 2 possible values $(0, 1)$, y also take 2 possible values and z also takes 2 possible values. Thus, there are $2 \times 2 \times 2 = 8$ possible choices for (x, y, z) and hence 8 divisors. Thus, we get a polynomial of degree 7. The corresponding polynomial is $1+2x+7x^2+14x^3+17x^4+34x^5+119x^6+238x^7$.

Another possible value of a_n can be $2 \times 238 = 2^2 \times 7 \times 17$ which has $3 \times 2 \times 2 = 12$ divisors as the prime factor 2 can take 3 values i.e. $0, 1, 2$. We get a polynomial of degree 11 in this case. The curious student may compute the divisors of 476 as $476 = 1 \times 476 = 2 \times 238 = 4 \times 119 = 7 \times 68 = 14 \times 34 = 17 \times 28$. The corresponding polynomial is

$1+2x+4x^2+7x^3+14x^4+17x^5+28x^6+34x^7+68x^8+119x^9+238x^{10}+476x^{11}$.

Another possible value of a_n is $3 \times 238 = 2 \times 3 \times 7 \times 17 = 714$ which has $2 \times 2 \times 2 \times 2 = 16$ divisors, giving a polynomial of degree 15. Also, a_n can be $4 \times 238 = 2^3 \times 7 \times 17 = 952$ which has $4 \times 2 \times 2 = 16$ divisors, giving a polynomial of degree 15. Next $a_n = 5 \times 238 = 1190 = 5 \times 2 \times 7 \times 17$ which has 16 divisors. Next, $a_n = 6 \times 238 = 2^2 \times 3 \times 7 \times 17 = 1428$ which has $3 \times 2 \times 2 \times 2 = 24$ divisors. $a_n = 7 \times 238 = 2 \times 7^2 \times 17 = 1666$ will have $2 \times 3 \times 2 = 12$ divisors. $a_n = 8 \times 238 = 2^4 \times 7 \times 17 = 1904$ has $5 \times 2 \times 2 = 20$ divisors. Next, $a_n = 9 \times 238 = 2142 > 2025$. Thus, the largest possible degree of such polynomials is 23. Please note that it is not necessary to construct the actual polynomial. We can just count the number of divisors.

16. A 100 digit number is formed by extracting any 100 consecutive digits after the decimal in the non-terminating decimal representation of $\frac{1}{7}$. What is the least possible sum of the digits of such a number ?

17 ৰ অসমাপ্ত দশমিক প্ৰকাশত দশমিক বিন্দুৰ পিছত 100 টা অংক ধাৰাবহিকভাৱে উলিয়াই এটি 100 অংকীয়া সংখ্যা গঠন কৰা হ'ল । এনে এটি সংখ্যাৰ অংক সমূহৰ সম্ভৱপৰ সৰ্বনিম্ন যোগফল কিমান ?

Ans : We have $\frac{1}{7} = 0.142857142857142857142857\ldots\ldots$. Thus, the group of six consecutive digits 142857 keeps on repeating. So, in the 100 consecutive digits that are extracted, there will be 16 such groups of 142857 together with 4 extra digits as $100 = 16 \times 6 + 4$. So, the sum of the digits of the 100 digit number formed is $16(1 + 4 + 2 + 8 + 5 + 7)+$ (Sum of those 4 extra digits) i.e. $432+$ (Sum of those 4 extra digits). There are several cases depending on the first digit of the 100 digit number.

(a) If the first digit is 1, then 142857 repeats 16 times and the last 4 digits are 1428. Required sum of digits is $432 + 1 + 4 + 2 + 8 = 447$.

(b) If the first digit is 4, then 428571 repeats 16 times and the last 4 digits are 4285. Required sum of digits is $432 + 4 + 2 + 8 + 5 = 451$.

(c) If the first digit is 2, then 285714 repeats 16 times and the last 4 digits are 2857. Required sum of digits is $432 + 2 + 8 + 5 + 7 = 454$.

(d) If the first digit is 8, then 857142 repeats 16 times and the last 4 digits are 8571. Required sum of digits is $432 + 8 + 5 + 7 + 1 = 453$.

(e) If the first digit is 5, then 571428 repeats 16 times and the last 4 digits are 5714. Required sum of digits is $432 + 5 + 7 + 1 + 4 = 449$.

(f) If the first digit is 7, then 714285 repeats 16 times and the last 4 digits are 7142. Required sum of digits is $432 + 7 + 1 + 4 + 2 = 446$.

Thus, the least possible sum of the digits is 446.

17. Let R be the set of real numbers and $f : \mathrm{R} \to \mathrm{R}$ be a function satisfying $f(x)f(y) = f(x - y)$ for all $x, y \in \mathrm{R}$. Find all possible values of $f(2025)$.

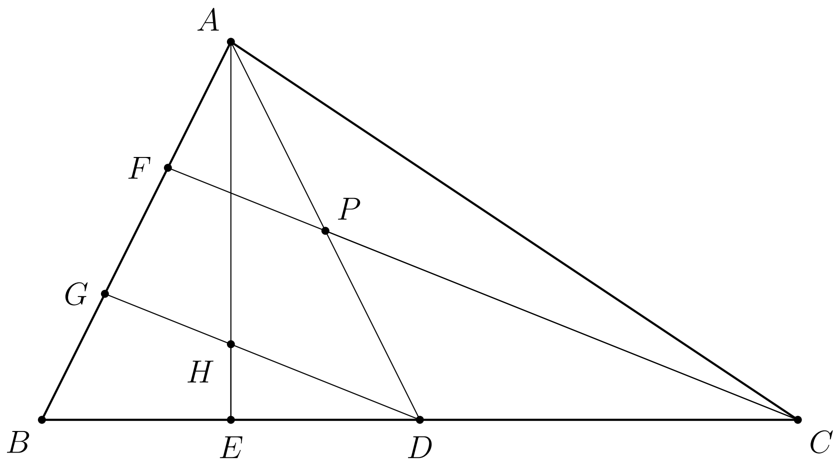
ধৰা হ'ল R বাস্তৱ সংখ্যাৰ সংহতি আৰু $f : \mathrm{R} \to \mathrm{R}$ এটি ফলন যাতে $f(x)f(y) = f(x - y)$, $x, y \in \mathrm{R}$ । $f(2025)$ ৰ আটাইবোৰ সম্ভৱপৰ মান উলিওৱা ।

Ans : Substituting $x = 0$ and $y = 0$ in $f(x)f(y) = f(x-y)$, we get $f(0)f(0) = f(0) \Rightarrow f(0) = 0$ or 1. If $f(0) = 0$, then for any given $x \in \mathrm{R}$, substituting $y = x$ gives $(f(x))^2 = f(0) = 0 \Rightarrow f(x) = 0$. Thus, $f(2025) = 0$.

If $f(0) = 1$, then for any given $x \in \mathrm{R}$, substituting $y = x$ gives $f(x)f(x) = f(0)$ i.e. $(f(x))^2 = 1$. Thus, either $f(x) = 1$ or $f(x) = -1$. However, substituting $x = 0$, we get $f(0)f(y) = f(0 - y)$ i.e. $f(y) = f(-y)$. So, $f(x - y) = f(x)f(y) = f(x)f(-y) = f(x - (-y)) = f(x + y)$. This gives $f(x + y) = f(x - y)$ for all x, y. Observe that for any $x \in \mathrm{R}$, $f(x) = f\left(\frac{x}{2} + \frac{x}{2}\right) = f\left(\frac{x}{2} - \frac{x}{2}\right) = f(0) = 1$. Thus, the only possible value of $f(x)$ in this case is 1. It follows that $f(2025) = 1$. Thus, the only possible values of $f(2025)$ are 0 and 1.

18. In the figure that follows, $BD = CD$, $BE = DE$, $AP = PD$ and $DG \parallel CF$.
Determine the ratio : $\dfrac{area\ (\triangle ADH)}{area\ (\triangle ABC)}$

তলত দিয়া চিত্ৰত, $BD = CD$, $BE = DE$, $AP = PD$ আৰু $DG \parallel CF$ । নিম্নোক্ত অনুপাতটো নিৰ্ণয় কৰা - $\dfrac{area\ (\triangle ADH)}{area\ (\triangle ABC)}$



Ans : As shown in figure,
Let, $BD = DC = x$ (say). Then, $BE = ED = \frac{x}{2}$. Now, P is the mid-point of AD and $PF \parallel DG$. So, F is the mid-point of AG and $PF = \frac{1}{2}GD$.

$$\therefore AF = FG \tag{1}$$

Again, D is the mid-point of BC and $DG \parallel CF$. So, G is the mid-point of BF and $GD = \frac{1}{2}CF$

$$\therefore BG = GF \tag{2}$$

From (1) and (2), we get:

$$AF = FG = BG \tag{3}$$

Similarly,

$$AK = KH \quad and \quad KP = \frac{1}{2}HD \tag{4}$$

Now, In $\triangle KCE$ and $\triangle HDE$

$$\angle HDE = \angle KCE \quad (corresponding\ angles)$$

$$\angle DHE = \angle CKE \quad (corresponding\ angles)$$

$$\therefore \triangle KCE \sim \triangle HDE \quad (AA\ rule)$$

$$\therefore \frac{KE}{HE} = \frac{CE}{DE} = \frac{x + \frac{x}{2}}{\frac{x}{2}} = 3$$

$$\Rightarrow KE = 3HE \tag{5}$$

From equation (5), we get:
$KH = 2HE$

$$\therefore AK = KH = 2HE \tag{6}$$

$$\therefore AE = AK + KH + HE$$

$$\Rightarrow AE = 2HE + 2HE + HE = 5HE$$

$$\therefore AE = 5HE \tag{7}$$

And,

$$AH = AK + KH = 2HE + 2HE = 4HE \tag{8}$$

Now,

$$\frac{ar\ (\triangle AHD)}{ar\ (\triangle AED)} = \frac{AH}{AE} = \frac{4HE}{5HE} = \frac{4}{5} \quad (using\ (7)\ and\ (8))$$

Also,

$$\frac{ar\ (\triangle AED)}{ar\ (\triangle ABC)} = \frac{ED}{BC} = \frac{\frac{x}{2}}{2x} = \frac{1}{4}$$

$$\therefore \frac{ar\ (\triangle AHD)}{ar\ (\triangle AED)} = \frac{ar\ (\triangle AHD)}{\frac{1}{4} ar\ (\triangle ABC)} = \frac{4}{5}$$

$$\Rightarrow \frac{ar\ (\triangle AHD)}{ar\ (\triangle ABC)} = \frac{1}{5} = 1:5$$