

Assam Academy of Mathematics
Assam Mathematics Olympiad 2023
Category II (Classes VII - VIII)

Full marks : 100

Time : 3 hours

There are 18 questions. Questions 1 to 5 carry 2 marks each. Questions 6 to 13 carry 5 marks each. Questions 14 to 18 carry 10 marks each. (Correction : There was a typo. Any five questions from 14 to 19 may be answered.)

ইয়াত 18 টা প্ৰশ্ন আছে। 1 ৰ পৰা 5 লৈ প্ৰতিটো প্ৰশ্নত 2 নম্বৰকৈ আছে। 6 ৰ পৰা 13 লৈ প্ৰতিটো প্ৰশ্নত 5 নম্বৰকৈ আছে। আৰু 14 ৰ পৰা 18 লৈ প্ৰতিটো প্ৰশ্নত 10 নম্বৰকৈ আছে।

There may be various other ways of solutions than those shown here. Queries or suggestions regarding the solutions can be mailed to mail@aamonline.in

ইয়াত দেখুওৱা ধৰণবিলাকৰ বাহিৰেও প্ৰশ্নবোৰৰ সমাধানৰ আন বিভিন্ন উপায় থাকিব পাৰে। সমাধানবোৰৰ বিষয়ে কিবা প্ৰশ্ন বা পৰামৰ্শ থাকিলে mail@aamonline.in লৈ মেইল কৰিব পাৰে।

1. Find the highest common factor of 131313, 232323, 333333, 434343.

131313, 232323, 333333 আৰু 434343 ৰ গৰিষ্ঠ সাধাৰণ উৎপাদক নিৰ্ণয় কৰা।

Ans : Observe that $131313 = 130000 + 1300 + 13 = 13 \times (10000 + 100 + 1) = 13 \times 10101$. Similarly, $232323 = 23 \times 10101$, $333333 = 33 \times 10101$ and $434343 = 43 \times 10101$. Since 13, 23, 33 and 43 do not have any other common factors, so the HCF is 10101.

2. Nina bought a notebook containing 96 pages and numbered on both sides from 1 to 192. Sumit tore out 20 pages of Nina's notebook and added the page numbers he found on these torn pages. Is it possible that Sumit got 2023 as the sum? Give reasons to your answer. (192 was wrongly printed as 2023 in the original question. The correction was communicated in the exam halls)

নীনাই 96 টা পৃষ্ঠা থকা বহী এখন কিনিছে, যাৰ পৃষ্ঠাসমূহৰ দুয়োটা পিঠিতেই 1 ৰ পৰা 2023 ৰ ভিতৰত এটাকৈ নম্বৰ(সংখ্যা) দি গ'ল। সুমিতে নীনাৰ বহীখনৰ 20 টা পৃষ্ঠা ফালি পেলালে আৰু সেই পৃষ্ঠাসমূহত পোৱা সংখ্যাসমূহ যোগ কৰিলে। যোগফল হিচাপে সুমিতে 2023 সংখ্যাটো পোৱা সম্ভৱনে? কাৰণ দৰ্শোৱা।

Ans : Each page has odd page number on one side and even on the other. So, sum of the page numbers on each page is odd. So, sum of the page numbers in 20 pages is even as the sum of even number of odd numbers is even. So, the sum can never be 2023.

3. Let $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$, for natural number n . Find the value of $(S_{10} - S_9) + (S_9 - S_8) + (S_8 - S_7)$.

ধৰা হ'ল, কোনো স্বাভাৱিক সংখ্যা n ৰ বাবে $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$. তেন্তে $(S_{10} - S_9) + (S_9 - S_8) + (S_8 - S_7)$ ৰ মান নিৰ্ণয় কৰা।

Ans :

$$\begin{aligned} & (S_{10} - S_9) + (S_9 - S_8) + (S_8 - S_7) \\ &= S_{10} - S_7 \\ &= \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{10}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{7}\right) \\ &= \frac{1}{8} + \frac{1}{9} + \frac{1}{10} = \frac{121}{360} \end{aligned}$$

4. Given that the number 148101a095 is divisible by 11, where a is some digit, what are the possible values of a ?

কোনো অংক a ৰ বাবে 148101a095 সংখ্যাটো 11 ৰে বিভাজ্য বুলি দিয়া আছে। a ৰ সম্ভৱপৰ মান কি কি হ'ব পাৰে?

Ans : If 148101a095 is divisible by 11, then the difference between the sum of digits in even places and the sum of digits in odd places should be divisible by 11. So, $(1 + 8 + 0 + a + 9) - (4 + 1 + 1 + 0 + 5) = 7 + a$ is divisible by 11. Hence, $7 + a = 0$ or 11 or 22 or \dots but a is a digit, so the only option is $7 + a = 11$ i.e. $a = 4$.

5. If each side of a cube is increased by 50% then by what percentage the surface area of the cube is increased?

যদি কোনো এটা ঘনকৰ প্ৰতিটো বাহুৰ দৈৰ্ঘ্য 50% বৃদ্ধি কৰা হয়, তেন্তে ঘনকটোৰ পৃষ্ঠকালি কিমান শতাংশ বৃদ্ধি পাব?

Ans : Let, a be length of each side of the cube. a' be the new increased length of each side of the cube. By question $a' = a + 50\%$ of $a = a + \frac{a}{2} = \frac{3a}{2}$. Original surface area is $6a^2$. New surface area is $6 \times \left(\frac{3a}{2}\right)^2 = \frac{27}{2}a^2$. So, percentage increase is $\frac{\frac{27}{2}a^2 - 6a^2}{6a^2} \times 100\% = 125\%$.

6. Let $p(x) = a + bx + cx^2 + dx^3 + ex^4$ be a polynomial where a, b, c, d, e are non-composite positive integers such that $a \times b \times c \times d \times e = 2023$. Find the value of $p(1)$.

ধৰা হ'ল $p(x) = a + bx + cx^2 + dx^3 + ex^4$ এটা বহুপদ, য'ত a, b, c, d, e অ-যৌগিক ধনাত্মক অখণ্ড সংখ্যা যাতে $a \times b \times c \times d \times e = 2023$ । তেন্তে $p(1)$ ৰ মান নিৰ্ণয় কৰা।

Ans : Given $a \times b \times c \times d \times e = 2023 = 7 \times 17 \times 17$. Since a, b, c, d, e are non-composite, so three of them are 7, 17, 17 and the remaining two are equal to 1. So, $p(1) = a + b + c + d + e = 7 + 17 + 17 + 1 + 1 = 43$.

7. The product of two numbers is 49280 and their HCF is 16. Find all such pairs of numbers.

দুটা সংখ্যাৰ পূৰণফল 49280 আৰু সিহঁতৰ গসাউ 16। তেনেকুৱা আটাইবোৰ সংখ্যা নিৰ্ণয় কৰা।

Ans : We know, product of two numbers is equal to the product of their HCF and LCM. Given, product is 49280 and HCF is 16, so the LCM is $49280/16 = 3080$. Now, $3080 = 8 \times 385$ which shows that LCM is not divisible by 16 i.e. the HCF. This contradicts that LCM is always divisible by HCF. Thus, there cannot exist any such pairs of numbers whose HCF is 16 and prouct is 49280.

8. Rahul had to catch a bus that was 225 m ahead of him. The bus also started at the same time. If the speed of the bus was 2.5 m/s and Rahul's speed was 36 km/h, in how much time can he catch the bus?

ৰাহুলে নিজতকৈ 225 মিঃ আগত থকা বাছ এখনত উঠিব লাগে। যিসময়ত ৰাহুলে খোজ কঢ়া আৰম্ভ কৰিছে, সেই সময়তে বাছখনেও যাবলৈ আৰম্ভ কৰিছে। যদি বাছখনৰ বেগ 2.5 মিঃ/চেকেণ্ড আৰু ৰাহুলৰ বেগ 36 কিঃমিঃ/ঘণ্টা হয়, তেন্তে কিমান সময়ৰ পাছত ৰাহুলে বাছখন ধৰিব পাৰিব ?

Ans : Speed of bus is $2.5m/s =$. Speed of Rahul is $36km/h = \frac{36000}{3600}m/s = 10m/s$. Let Rahul catch the bus in time t seconds. Distance travelled by Rahul in time t is $10t$ and distance travelled by bus in time t is $2.5t$. By question, since they started together, $10t - 2.5t = 225$ i.e. $7.5t = 225$ i.e. $t = 30$.

9. A can do a piece of work in 25 days and B can finish it in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the remaining work?

A য়ে এটা কাম 25 দিনত কৰিব পাৰে আৰু B য়ে সেই কামটো 20 দিনত কৰিব পাৰে। তেওঁলোক দুয়ো 5 দিন একেলগে কাম কৰে আৰু তাৰ পাছত A য়ে গুচি যায়। বাকী থকা কামখিনি Bয়ে কিমানদিনত শেষ কৰিব পাৰিব?

Ans : In 1 day, A can do $\frac{1}{25}$ of the work. In 1 day, B can do $\frac{1}{20}$ of the work. Together, in 1 day they can do $\frac{1}{25} + \frac{1}{20} = \frac{9}{100}$ of the work. In 5 days, they can do $\frac{5 \times 9}{100} = \frac{9}{20}$ of the work. Thus, part of work remaining after 5 days is $1 - \frac{9}{20} = \frac{11}{20}$. This is to be done by B alone. Complete work can be done by B in 20 days. So, $\frac{11}{20}$ of the work can be done in $20 \times \frac{11}{20} = 11$ days.

10. If a, b, c, d satisfy the equations $a + 7b + 3c + 5d = 0$, $8a + 4b + 6c + 2d = -16$, $2a + 6b + 4c + 8d = 16$ and $5a + 3b + 7c + d = -16$. Find the value of $(a + d)(b + c)$.

যদি a, b, c, d য়ে $a + 7b + 3c + 5d = 0$, $8a + 4b + 6c + 2d = -16$, $2a + 6b + 4c + 8d = 16$ আৰু $5a + 3b + 7c + d = -16$ এইকেইটা সমীকৰণ সিদ্ধ কৰে। তেন্তে $(a + d)(b + c)$ ৰ মান নিৰ্ণয় কৰা।

Ans : Given

$$a + 7b + 3c + 5d = 0$$

$$8a + 4b + 6c + 2d = -16$$

$$2a + 6b + 4c + 8d = 16$$

$$5a + 3b + 7c + d = -16$$

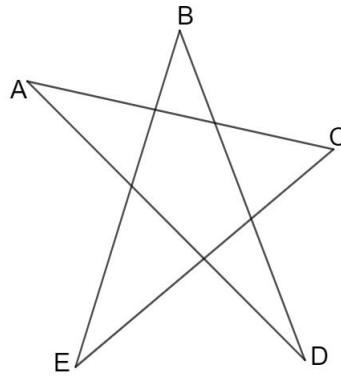
Adding the 2nd and 3rd equations, we get

$$10(a + b + c + d) = 0 \text{ i.e. } a + b + c + d = 0.$$

Adding all the equations, we get $16(a+d) + 20(b+c) = -16$. Now, $a+b+c+d = 0$, so $a+d = -(b+c)$. Thus, $-16(b+c) + 20(b+c) = -16$, so that $b+c = -4$. Also, $a+d = -(b+c) = 4$. Hence $(a+d)(b+c) = -16$.

11. In the following star, find the sum of the angles A, B, C, D and E .

তলৰ তৰাঙ্কেত্ৰটোত A, B, C, D আৰু E কোণৰ যোগফল নিৰ্ণয় কৰা।



Ans : We label the angles in the star as in the diagram.

By angle sum property of triangles, we have

$A + a_1 + a_2 = B + b_1 + b_2 = C + c_1 + c_2 = D + d_1 + d_2 = E + e_1 + e_2 = 180^0$.
Now, the angles a_1, y and a_2, u form linear pairs. So, $a_1 = (180^0 - y)$ and $a_2 = (180^0 - u)$, so that $a_1 + a_2 = 360^0 - (u + y)$. Similarly, $b_1 + b_2 = 360^0 - (u + v)$, $c_1 + c_2 = 360^0 - (v + w)$, $d_1 + d_2 = 360^0 - (w + x)$ and $e_1 + e_2 = 360^0 - (x + y)$.

$$\begin{aligned} A + a_1 + a_2 &= 180^0 \\ \Rightarrow A + 360^0 - (u + y) &= 180^0 \\ \Rightarrow A &= u + y - 180^0 \end{aligned}$$

Similarly, $B = u + v - 180^0$, $C = v + w - 180^0$, $D = w + x - 180^0$ and $E = x + y - 180^0$. Thus, $A + B + C + D + E = 2(u + v + w + x + y) - 5 \times 180^0$. By angle sum property of pentagon, $u + v + w + x + y = 540^0$. So, $A + B + C + D + E = 2 \times 540^0 - 5 \times 180^0 = 1080^0 - 900^0 = 180^0$.

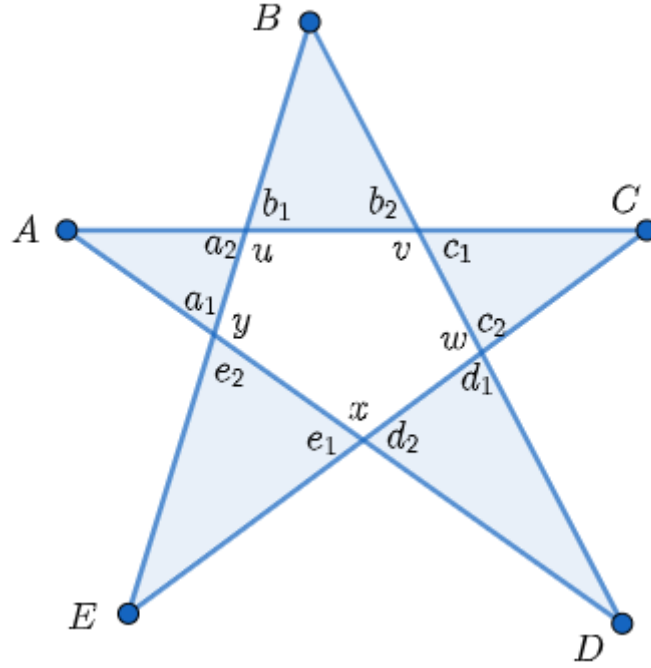


Figure 1: Star

12. Find the smallest number greater than 2023 which when divided by 6, 12, 68 and 102 leaves remainder 5 in each case.

এনেকুরা আটাইতকৈ সৰু সংখ্যাটো নিৰ্ণয় কৰা যিটো 2023 তকৈ ডাঙৰ আৰু যাক 6, 12, 68 আৰু 102 ৰে হৰণ কৰিলে প্ৰতিটো ক্ষেত্ৰতে ভাগশেষ 5 থাকে।

Ans : LCM of 6, 12, 68 and 102 is 204. The subsequent common multiples are $204 \times 2, 204 \times 3, 204 \times 4, \dots$ and so on. Thus, the smallest common multiple of 6, 12, 68 and 102 that is greater than 2023 is $204 \times 10 = 2040$. Required number is $2040 + 5 = 2045$.

13. Three fourth of a tank is filled with water. A pipe can fill it in 12 minutes and another pipe can empty it in 8 minutes. If both the pipes are opened together, find the time in which the tank will get emptied ?

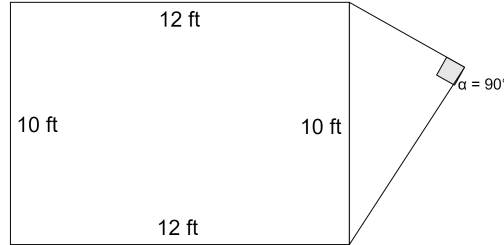
এটা টেংকীৰ তিনি চতুৰ্থাংশ পানী ৰে ভৰি আছে। এডাল পাইপে 12 মিনিটত ইয়াক পানীৰে সম্পূৰ্ণ কৰিব পাৰে আৰু আন এডালে ইয়াক 8 মিনিটত সম্পূৰ্ণৰূপে খালী কৰিব পাৰে। যদি দুয়োডাল পাইপ একেলগে খুলি দিয়া হয়, তেন্তে টেংকীটো খালী হৈ পৰিবলৈ কিমান সময় লাগিব সেয়া নিৰ্ণয় কৰা।

Ans : In one minute, the pipe can fill $\frac{1}{12}$ part of the tank. In one minute, the other pipe can empty $\frac{1}{8}$ part of the tank. Thus, effectively, in one minute, $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$ of the tank is emptied. So, the full tank can be emptied in 24 minutes. Hence, three fourth of the tank can be emptied in $\frac{3}{4} \times 24 = 18$ minutes.

14. A rectangular room has a balcony attached externally to its shorter side in form of a right angled triangle. The sides of the room are of lengths 12 ft

and 10 ft and the sides of the balcony are integers in ft. How many square feet of carpet is required to cover the floor of the room with the balcony ?

এটা আয়তাকাৰ কোঠাৰ চুটি বাহুটোৰ সৈতে বাহিৰৰ পৰা সংলগ্ন সমকোণী ত্ৰিভুজাকৃতিৰ বাৰাণ্ডা এখন আছে। কোঠাটোৰ বাহুদুটাৰ দৈৰ্ঘ্য 12 ফুট আৰু 10 ফুট আৰু বাৰাণ্ডাখনৰ বাহুসমূহৰ দৈৰ্ঘ্যও অখণ্ড সংখ্যা (ফুট এককত)। বাৰাণ্ডাখনৰ সৈতে কোঠাটোৰ মজিয়াখন আঙুৰিবলৈ কিমান বৰ্গফুট কাৰ্পেটৰ প্ৰয়োজন হ'ব?



Ans : Since the sides of the balcony are integers and $10^2 = 8^2 + 6^2$, so the right triangular balcony has the perpendicular sides of lengths 8 cm and 6 cm. Area of the room is 120 sq. ft. Area of the balcony is $\frac{1}{2} \times 8 \times 6 = 24$ sq. ft. Thus total carpet required is 144 sq. ft.

15. Eight cards have numbers 1,2,3,4,5,6,7,8 written on them. They are rearranged in various ways and placed one over the other. In how many cases will the top two cards form a pair of twin primes i.e. primes which differ by two ?

আঠখন পাতত 1,2,3,4,5,6,7,8 এই সংখ্যাসমূহ লিখা আছে। সিহঁতক বিভিন্নধৰণে পুনৰ্বিন্যস্ত কৰা হ'ল আৰু এখনৰ ওপৰত এখনকৈ ৰখা হ'ল। কিমান ক্ষেত্ৰত একেবাৰে ওপৰৰ পাতদুখনত থকা সংখ্যা দুটাই যুগ্ম মৌলিক সংখ্যাৰ (অৰ্থাৎ এনেকুৱা মৌলিক সংখ্যা যাৰ মাজত পাৰ্থক্য 2) যোৰ এটা গঠন কৰিব?

Ans : Total number of arrangements is $8! = 40320$. The only pairs of twin primes here are 3, 5 and 5, 7. Number of arrangements where 3, 5 are top two cards is $6! \times 2! = 720 \times 2 = 1440$. Number of arrangements where 5, 7 are top two cards is $6! \times 2! = 720 \times 2 = 1440$. So, required number of arrangements where the top two cards form a pair of twin primes is $1440 + 1440 = 2880$.

16. Let M be the intersection point of medians of a triangle $\triangle ABC$. Given $AC = 2BC$ and $\angle ACM = \angle CBM$. Find $\angle ACB$.

ধৰা হ'ল M হৈছে $\triangle ABC$ ৰ মধ্যমাকেইডালৰ ছেদবিন্দু। দিয়া আছে $AC = 2BC$ আৰু $\angle ACM = \angle CBM$ । তেন্তে $\angle ACB$ ৰ মান নিৰ্ণয় কৰা।

Ans : Since BD is median, so D is the mid-point of AC. So, let $AD = DC = \frac{AC}{2} = BC = x$. Given, $\angle ACM = \angle CBM = \alpha$, say. Since $DC = BC = x$, so we have $\angle CDM = \angle CBM = \alpha$. Thus, in triangle CDM, $CM = MD = y$ say. Since BD is a median and M is the centroid, so $BM : MD = 2 : 1$. Hence, $BM = 2y$.

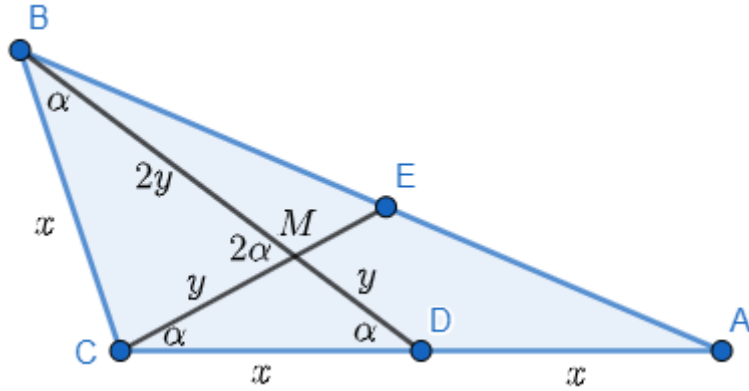


Figure 2: Labelled diagram

Now, $\triangle CDM$ is similar to $\triangle BCD$. So, their corresponding sides are proportional. Hence,

$\frac{CM}{BC} = \frac{MD}{DC} = \frac{DC}{BD} \Rightarrow \frac{y}{x} = \frac{y}{x} = \frac{x}{2y+y} \Rightarrow x^2 = 3y^2$. Thus, in $\triangle BMC$, $BC^2 + CM^2 = x^2 + y^2 = 3y^2 + y^2 = (2y)^2 = BM^2$. So, $\triangle BMC$ is a right angle triangle with $\angle BCM = 90^\circ$ which gives $\alpha + 2\alpha = 90^\circ$ i.e. $\alpha = 30^\circ$. Thus, $\angle ACB = \angle ACM + \angle BCM = \alpha + 90^\circ = 120^\circ$.

17. Find the number of non-negative integer solutions of $x^2 + y = 40320$. Also, list all the solutions.

$x^2 + y = 40320$ সমীকরণটোর কিমানটা অঋণাত্মক অখণ্ড সমাধান থাকিব? সমাধানসমূহ লিখি উলিওরা।

Ans : Starting with $x = 0$ and proceeding with consecutive integers we get the non-negative integer solutions as $x = 0, y = 40320$; $x = 1, y = 40319$; $x = 2, y = 40316$; $x = 3, y = 40311$ and so on. We shall stop when x^2 takes the value of the perfect square just smaller than 40320. It is easily observed that $200^2 = 40000$ and $201^2 = 40401 > 40320$. Thus, there are exactly 201 non-negative integer solutions given by $x = n, y = 40320 - n^2$ where $n = 0, 1, 2, \dots, 200$.

18. Evaluate $\sqrt{2023 \times 2024 \times 2025 \times 2026 + 1}$.

$\sqrt{2023 \times 2024 \times 2025 \times 2026 + 1}$ ৰ মান নির্ণয় কৰা।

Ans : Consider $\sqrt{n(n+1)(n+2)(n+3) + 1}$.

$$\begin{aligned} & \sqrt{n(n+1)(n+2)(n+3) + 1} \\ &= \sqrt{(n^2 + 3n)(n^2 + 3n + 2) + 1} \\ &= \sqrt{(n^2 + 3n)^2 + 2(n^2 + 3n) + 1} \\ &= \sqrt{(n^2 + 3n + 1)^2} \\ &= n^2 + 3n + 1 \end{aligned}$$

Putting $n = 2023$, $\sqrt{2023 \times 2024 \times 2025 \times 2026 + 1} = 2023^2 + 3 \times 2023 + 1 = 4098599$.

19. A student starts printing the natural numbers, one in each page, in the natural order. He stops when 3333 digits have been printed in all. How many pages did the student print ?

এজন ছাত্রই এখন পৃষ্ঠাত এটাকৈ স্বাভাৱিক সংখ্যা, স্বাভাৱিক ক্ৰমত ছপা কৰা আৰম্ভ কৰে। যেতিয়া মুঠ 3333 টা অংক ছপা হয়, তেতিয়া ছাত্রজনে ছপা কৰা বন্ধ কৰে। তেওঁ মুঠতে কিমানটা পৃষ্ঠাত সংখ্যা ছপা কৰিছিল?

Ans : Total number of one-digit numbers printed = 9 (From 1 - 9). Number of digits used = 9.

Total number of two-digit numbers printed = 90 (From 10 - 99). Number of digits used = $2 \times 90 = 180$

Total number of three-digit number is 900 (From 100 - 999)

Number of digits used = $3 \times 900 = 2700$

Total number of digits used so far = $9 + 180 + 2700 = 2889$

Number of digits remaining = $3333 - 2889 = 444$.

Number of four-digit numbers required to be used = $\frac{444}{4} = 111$.

So, four-digit numbers are used from 1000 to 1110. The student prints 1110 pages.