

Assam Academy of Mathematics  
Assam Mathematics Olympiad 2022  
Category I (Classes V - VI)  
21st August 2022

Full marks : 100

Time : 3 hours

There are 18 questions. Questions 1 to 5 carry 2 marks each. Questions 6 to 13 carry 5 marks each. Questions 14 to 18 carry 10 marks each.

ইয়াত 18 টা প্ৰশ্ন আছে। 1 ৰ পৰা 5 লৈ প্ৰতিটো প্ৰশ্নত 2 নম্বৰকৈ আছে। 6 ৰ পৰা 13 লৈ প্ৰতিটো প্ৰশ্নত 5 নম্বৰকৈ আছে। আৰু 14 ৰ পৰা 18 লৈ প্ৰতিটো প্ৰশ্নত 10 নম্বৰকৈ আছে।

There may be various other ways of solutions than those shown here. Queries or suggestions regarding the solutions can be mailed to mail@aamonline.in

1. You are given the digits 1, 2, 3, 4, 5, 6, 7, 8. What are the smallest and greatest numbers of 6 digits that you can form with these digits without repeating any of the digits? If you are allowed to repeat the digits, what are the smallest and the greatest numbers of 6 digits that you can form using these digits?

তোমাক 1, 2, 3, 4, 5, 6, 7, 8 অংককেইটা দিয়া হৈছে। এটাও অংক পুনৰাবৃত্তি নকৰাকৈ এই অংককেইটাৰ পৰা 6 টা অংক লৈ গঠন কৰিব পৰা আটাইতকৈ সৰু আৰু ডাঙৰ সংখ্যা দুটা কি কি? যদি তুমি অংকবোৰ পুনৰাবৃত্তি কৰিব পাৰা, তেন্তে এই অংককেইটাৰ পৰা 6 টা অংক লৈ গঠন কৰিব পৰা আটাইতকৈ সৰু আৰু ডাঙৰ সংখ্যা দুটা কি কি?

Ans : 123456, 876543, 111111 and 888888 respectively.

2. A tank is full of 300 litres of water. How much water will be there in 25 such full tanks? If 15 buckets can be filled with one tank of water, how many buckets in all can be filled with the water in 25 tanks?

এটা টেংকত 300 লিটাৰ পানী ধৰে। তেনেকুৱা 25 টা টেংকত কিমান পানী ধৰিব? যদি এটা টেংকৰ পানীৰে 15 টা বাল্টি ভৰাব পাৰি, তেন্তে 25 টা টেংকৰ পানীখিনি মুঠতে কিমানটা বাল্টিত ভৰাব পৰা যাব?

Ans : Amount of water in 25 tanks =  $300 \times 25 = 7500$  litres. Number of buckets that can be filled is  $15 \times 25 = 375$ .

3. What is the least positive integer by which  $2^3 \times 3^4 \times 4^5 \times 5^6 \times 6^7$  should be multiplied so that the product is a perfect square?

$2^3 \times 3^4 \times 4^5 \times 5^6 \times 6^7$  সংখ্যাটো পূৰ্ণবৰ্গ সংখ্যা হ'বলৈ হ'লে তাক পূৰ্ণ কৰিবলগীয়া আটাইতকৈ সৰু ধনাত্মক পূৰ্ণসংখ্যাটো কি?

Ans :  $2^3 \times 3^4 \times 4^5 \times 5^6 \times 6^7 = 2^3 \times 3^4 \times 2^{10} \times 5^6 \times 2^7 \times 3^7 = 2^{20} \times 3^{11} \times 5^6$ . For a perfect square, the powers of each prime factor should be even. So the least such positive integer is 3.

4. A man walks 200 metres from his home through his lane to the main road. Then, he takes an auto-rickshaw to travel 3 kilometres. He gets down and walks 300 metres to his friend's place. What is the total distance covered by the man in kilometres?

এজন মানুহে নিজৰ ঘৰৰ পৰা 200 মিটাৰ খোজ কাঢ়ি মূল পথ পালেগৈ। তাৰ পাছত, তেওঁ এখন অটো-ৰিক্সা লৈ 3 কিলোমিটাৰ যাত্ৰা কৰে। তাৰ পৰা তেওঁ 300 মিটাৰ খোজ কাঢ়ি বন্ধুৰ ঘৰলৈ পায়গৈ। মানুহজনে অতিক্ৰম কৰা মুঠ দূৰত্ব কিলোমিটাৰত প্ৰকাশ কৰা।

$$\text{Ans : } 200m + 3km + 300m = \left( \frac{200}{1000} + 3 + \frac{300}{1000} \right) km = (0.2 + 3 + 0.3)km = 3.5km.$$

5. Two circles are given to you. The area of the first circle is twice the second. If the radius of the first circle is  $r$ , what is the radius of the second circle ?

তোমাক দুটা বৃত্ত দিয়া হৈছে। প্ৰথম বৃত্তটোৰ ক্ষেত্ৰফল দ্বিতীয়টোৰ দুগুণ। যদি প্ৰথমটো বৃত্তৰ ব্যাসার্ধ  $r$  হয়, তেন্তে দ্বিতীয়টো বৃত্তৰ ব্যাসার্ধ কিমান হ'ব?

$$\text{Ans : } \text{Let the radius of the second circle be } s. \text{ By question, } \pi r^2 = 2\pi s^2 \Rightarrow s^2 = \frac{r^2}{2} \Rightarrow s = \frac{r}{\sqrt{2}}.$$

6. Three alarms ring at intervals of 10, 15 and 30 minutes respectively. If all three ring together at 6 am, how many times will they ring together till 10:45 am, counting from 6 am?

তিনিটা সংকেত (এলাৰ্ম) ক্ৰমে 10, 15 আৰু 30 মিনিটৰ ব্যৱধানত বাজি থাকে। যদি পুৱা 6 টাত তিনিওটা একেলগে বাজে, তেন্তে পুৱা 6 টাৰ সংকেতটোকো ধৰি পুৱা 10:45 টালৈকে কিমানবাৰ একেলগে বাজিব?

Ans : The bells will ring together after each LCM(10,15,30) minutes i.e. 30 minutes. Thus, they ring together at 6, 6:30, 7, 7:30, 8, 8:30, 9, 9:30, 10 and 10:30. That's 10 times in all.

7. If Figure A represents the following matrix, then what matrix will be represented by Figure B?

যদি চিত্ৰ A এ তলৰ মৌলকক্ষটো সূচায় তেন্তে চিত্ৰ B য়ে সূচাবলগীয়া মৌলকক্ষটো কি?

$$\begin{pmatrix} 3 & 2 & 1 & 1 \\ 2 & 1 & & \\ 1 & & & \end{pmatrix}$$

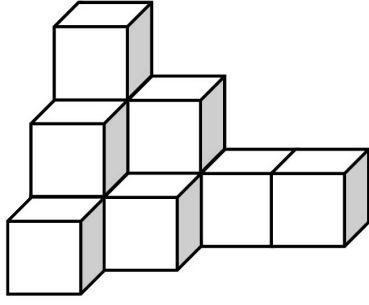


Figure A

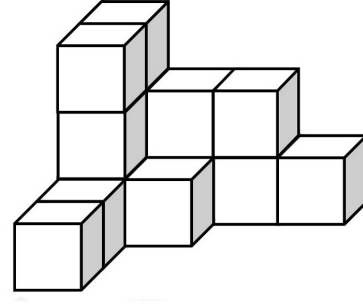


Figure B

Ans : Let us count the number of cubes. Note that some cubes are hidden from view. In Figure A, there are 3 cubes in the first column of the row that is farthest from view. There are 2 cubes in the second column of that row and 1 each in the third and fourth columns. This gives the first row in the matrix as 3, 2, 1, 1. Similarly, the second row is 2, 2 and the third is 1. Applying the same observation to Figure B, we see that the matrix should be

$$\begin{pmatrix} 3 & 2 & 2 & 1 \\ 3 & 1 & & \\ 1 & & & \\ 1 & & & \end{pmatrix}$$

8. List all possible positive factors (divisors) of 2022 and find their sum.

২০২২ ৰ সকলো ধনাত্মক উৎপাদক (ভাজক) লিখা আৰু সিহঁতৰ যোগফল নিৰ্ণয় কৰা।

Ans :  $2022 = 2 \times 3 \times 337$ . The factors are 1, 2, 3, 6, 337, 674, 1011 and 2022. Thus, the sum of all the factors is 4056.

9.  $abc$  is a three-digit number where  $a, b, c$  are digits. It is divisible by 5 but not by 2. The sum of its digits is 14. Find the sum of all such three digit numbers.

$abc$  এটা তিনিটা অংকৰে গঠিত সংখ্যা, য'ত  $a, b, c$  হৈছে দশমিক অংক। সংখ্যাটোক 5 ৰে হৰণ যায় কিন্তু 2 ৰে নাযায়। ইয়াৰ অংকবোৰৰ যোগফল 14। তিনিটা অংকৰে গঠিত এনেকুৱা আটাইকেইটা সংখ্যাৰ যোগফল নিৰ্ণয় কৰা।

Ans : Since  $abc$  is divisible by 5 but not by 2, so  $c = 5$ . Now,  $a + b + c = 14 \Rightarrow a + b = 9$ . Thus, all such numbers are 185, 275, 365, 455, 545, 635, 725, 815, 905. Their sum is 4905.

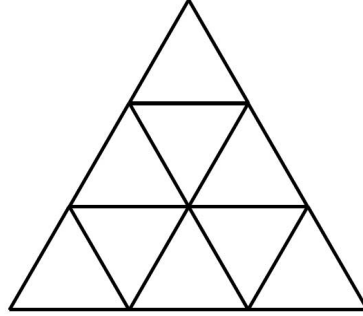
10. A father is 5 times older than his daughter. After 5 years, the sum of their ages will be 52. Find their present ages.

এজন পিতৃ বয়সত তেওঁৰ জীয়েকতকৈ 5 গুণ ডাঙৰ। 5 বছৰৰ পাছত তেওঁলোকৰ বয়সৰ যোগফল হ'ব 52। তেওঁলোকৰ বৰ্তমানৰ বয়স নিৰ্ণয় কৰা।

Ans : If the daughter's age is  $x$  then the father's age will be  $5x$ . By question  $(x + 5) + (5x + 5) = 52$ . So,  $x = 7$ .

11. In the following figure, the side length of the largest triangle is 3 units. If we take a similar figure where the side length of the largest triangle is 100 units, then how many triangle of side length 1 will be there?

তলৰ চিত্ৰত আটাইতকৈ ডাঙৰ ত্ৰিভুজটোৰ বাহুৰ দৈৰ্ঘ্য 3 একক। যদি আমি একেধৰণৰ এটা চিত্ৰ লওঁ, য'ত আটাইতকৈ ডাঙৰ ত্ৰিভুজটোৰ বাহুৰ দৈৰ্ঘ্য 100 একক, তেন্তে তাত 1 একক বাহু-দৈৰ্ঘ্যৰ ত্ৰিভুজ কিমানটা থাকিব?



Ans : If the side length of the largest triangle is 1 unit, then we get 1 triangle. If the side length of the largest triangle is 2 units, then we get  $1 + 3$  triangles. If the side length of the largest triangle is 3 units, then we get  $1 + 3 + 5$  triangles. Proceeding in this way, we get the required number of triangles is  $1 + 3 + 5 + \dots + (2 \times 100 - 1) = 100^2 = 10000$ . Another way of approach is to observe that each larger equilateral triangle is symmetrically divided into smaller equilateral triangles of side length 1. Area of equilateral triangle of side 100 is  $\frac{\sqrt{3}}{4} \times 100^2$ . Area of each equilateral triangle of side 1 is  $\frac{\sqrt{3}}{4}$ . Thus, the number of small equilateral triangles is  $100^2 = 10000$ .

12. A student computes the average of the first 100 natural numbers but mistakenly she took 35 as 25, 50 as 60 and 15 as 45. Compute the correct average and the average computed by the student. (Hint: Sum of the first  $n$  natural numbers is  $n(n + 1)/2$ .)

এগৰাকী ছাত্ৰীয়ে প্ৰথম 100 টা স্বাভাৱিক সংখ্যাৰ গড় নিৰ্ণয় কৰোঁতে ভুলতে 35 ক 25, 50 ক 60 আৰু 15 ক 45 বুলি ল'লে। সঠিক গড়টো আৰু ছাত্ৰীগৰাকীয়ে নিৰ্ণয় কৰা গড়টো নিৰ্ণয় কৰা। (সংকেত: প্ৰথম  $n$  টা স্বাভাৱিক সংখ্যাৰ যোগফল  $n(n + 1)/2$ )

Ans : Correct sum of 1st 100 natural numbers =  $\frac{100 \times (100 + 1)}{2} = 5050$ . So, correct average = 50.5. Let  $S$  be the wrong total computed by the student. Then,  $S = 5050 - 35 + 25 - 50 + 60 - 15 + 45 = 5080$ . So, the wrong average is 50.8.

13. Up to 1000 how many numbers can be expressed as a sum of two consecutive positive integers?

1000 লৈকে কিমানটা সংখ্যক দুটা ক্ৰমিক ধনাত্মক অখণ্ড সংখ্যাৰ যোগফল ৰূপে প্ৰকাশ কৰিব পাৰিব?

Ans : Two consecutive integers can be denoted as  $a$  and  $a + 1$ . Their sum is  $2a + 1$  which is always odd. Also, any odd positive integer can be written as  $2a + 1 = a + (a + 1)$ . So we just need to find the number of odd numbers from 1 to 1000 which is 500. But 1 cannot be written as  $2a + 1$  for positive  $a$  as in that case  $a = 0$ . So, the odd numbers 3, 5, 7, . . . . . 999 can be written as sum of two consecutive positive integers. They are 499 in all.

14. Find the gcd of the following numbers.

তলৰ সংখ্যাসমূহৰ গসাণ্ড (gcd) নিৰ্ণয় কৰা।

$$\begin{aligned} &1 \times 11 \times 21 \times \cdots \times 91, \\ &2 \times 12 \times 22 \times \cdots \times 92, \\ &3 \times 13 \times 23 \times \cdots \times 93, \\ &\cdots, \\ &9 \times 19 \times 29 \times \cdots \times 99, \\ &10 \times 20 \times 30 \times \cdots \times 100. \end{aligned}$$

Ans : There are 10 numbers given to us. We try to find the possible common factors of the numbers. Let us consider the following number

$$10 \times 20 \times 30 \times \cdots \times 100.$$

Since, 10 is not a factor of the remaining numbers, so 10 cannot be a common factor. We remove 10's as a factor from this number. Then we get the following number which may be a common factor

$$1 \times 3 \times 4 \times 6 \times 7 \times 8 \times 9.$$

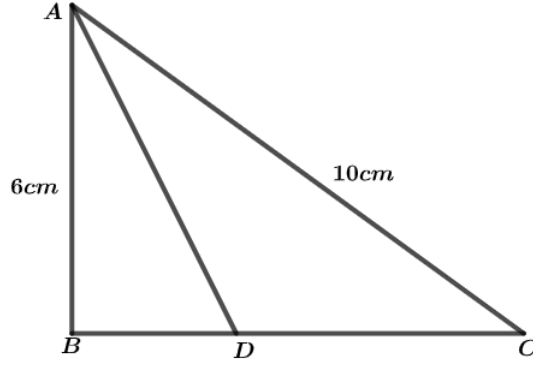
Again, some of the given numbers are odd. So, 2 cannot be a common factor. So, we can remove the factors from the above number which are powers of 2. So we are left with

$$3 \times 3 \times 7 \times 9 = 3^4 \times 7.$$

Now, we see that each given number is divisible by  $3^4$ . The respective factors 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 have  $3^4$  as a common divisor. Also, the respective factors 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49 have 7 as a common divisor. Hence the required gcd is  $3 \times 3 \times 7 \times 9 = 567$ .

15.  $ABC$  is a right angled triangle, right angled at  $B$ .  $AB = 6$  cm,  $AC = 10$  cm.  $D$  is a point on  $BC$  such that  $BD : DC = 3 : 5$ . Find the area of the triangle  $ADC$ .

$ABC$  এটা সমকোণী ত্ৰিভুজ, যাৰ  $B$  কোণটো এটা সমকোণ,  $AB = 6$  চেণ্টিমিটাৰ,  $AC = 10$  চেণ্টিমিটাৰ।  $D$  বিন্দুটো  $BC$  ৰ ওপৰত এনেদৰে আছে যাতে  $BD : DC = 3 : 5$ ।  $ADC$  ত্ৰিভুজটোৰ কালি উলিওৱা।



Ans : By Pythagoras Theorem,  $BC^2 = 10^2 - 6^2 = 64$ . So,  $BC = 8$ .  $DC = \frac{5}{8}BC = \frac{5}{8} \times 8 = 5$ . So, area of triangle  $ADC = \frac{1}{2}DC \times AB = \frac{1}{2} \times 5 \times 6 = 15$  sqcm.

16. A car travels 90 km in  $2\frac{1}{2}$  hours. Find the time required by the car to cover 30 km in the same speed. Also, find the distance covered by the car in 2 hours with the same speed.

এখন গাড়ীয়ে  $2\frac{1}{2}$  ঘণ্টাত 90 কিলোমিটাৰ যাত্ৰা কৰে। একে বেগত গাড়ীখনে 30 কিলোমিটাৰ যাবলৈ হ'লে কিমান সময় লাগিব নিৰ্ণয় কৰা। আকৌ, গাড়ীখনে একে বেগত 2 ঘণ্টাত কিমান দূৰত্ব যাব পাৰিব নিৰ্ণয় কৰা।

Ans :  $2\frac{1}{2}$  hours = 150 minutes. Time required to cover 90km is 150 minutes. So, time required to cover 1 km is  $\frac{150}{90}$  minutes. So, time required to cover 30km is  $\frac{150}{90} \times 30 = 50$  minutes.

Distance travelled in 150 minutes is 90km. So, distance travelled in 1 minute is  $\frac{90}{150}$  km. Hence, distance travelled in 120 minutes is  $\frac{90}{150} \times 120 = 72$ km.

17. You need to arrange the letters  $A, B, C, D$  in the boxes below without repeating any letter. For the Arrangement 1,  $A$  is fixed in the first place and for the Arrangement 2,  $X$  is fixed in the first place and  $Y$  is fixed in the third place. For example,  $ABDC$  is a type of Arrangement 1 and  $XAYC, XDYB$  are types of Arrangement 2. List all possible arrangements in both the cases.

তলৰ ঘৰকেইটাত  $A, B, C, D$  আখৰকেইটা পুনৰাবৃত্তি নকৰাকৈ সজাব লাগে। Arrangement 1 ত প্ৰথম স্থানটো  $A$  ৰ বাবে নিৰ্ধাৰিত কৰি থোৱা হৈছে। Arrangement 2 ত প্ৰথম স্থানত  $X$  আৰু তৃতীয় স্থানত  $Y$  নিৰ্ধাৰিত কৰি থোৱা হৈছে। উদাহৰণস্বৰূপ,  $ABDC$  ধৰণটো Arrangement 1 ত পৰে, আৰু  $XAYC, XDYB$  ধৰণ দুটা Arrangement 2 ত পৰে। দুয়োটা ক্ষেত্ৰৰে সম্ভাৱ্য আটাইবোৰ ধৰণ তালিকাভুক্ত কৰা।



Arrangement 1

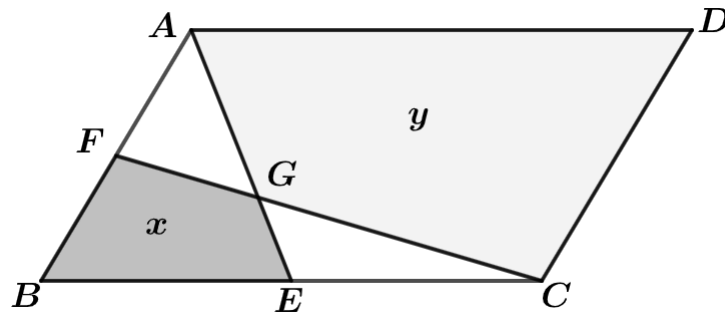


Arrangement 2

Ans : For Arrangement 1, the six possibilities are ABCD, ABDC, ACBD, ACDB, ADBC, ADCB. For Arrangement 2, the twelve possibilities are XAYB, XAYC, XAYD, XBYA, XBYC, XBYD, XCYA, XCYB, XCYD, XDYA, XDYB, XDYC.

18.  $ABCD$  is a parallelogram of area 180 sq.cm.  $E$  and  $F$  are the mid-points of  $BC$  and  $BA$  respectively. Area of the quadrilateral  $BFGE$  is  $x$  and that of  $ADCG$  is  $y$ . The area of the quadrilateral  $ADCG$  is four times that of  $BFGE$ . Find the values of  $x$  and  $y$ .

$ABCD$  সামান্তরিকটো কালি 180 বর্গ সেন্টিমিটার।  $E$  আৰু  $F$  ক্ৰমে  $BC$  আৰু  $BA$  ৰ মধ্যবিন্দু। চতুৰ্ভুজ  $BFGE$  ৰ কালি  $x$  আৰু  $ADCG$  ৰ কালি  $y$ । চতুৰ্ভুজ  $ADCG$  ৰ কালি  $BFGE$  কালিৰ চাৰি গুণ।  $x$  আৰু  $y$  ৰ মান নিৰ্ণয় কৰা।



Ans : Let area of the parallelogram  $ABCD$  be  $\Delta$ . Since  $E$  and  $F$  are the mid-points of  $BC$  and  $AB$  respectively, so  $\text{Area}(CBF) = \text{Area}(ABE) = \Delta/4$ . Combined area of the portion covered by these two triangles is  $\Delta/4 + \Delta/4 - x = \Delta/2 - x$ . Thus, the area  $y = \Delta - (\Delta/2 - x) = \Delta/2 + x$ . By question,  $y = 4x$ . So,  $\Delta/2 + x = 4x$  i.e.  $x = \Delta/6 = 180/6 = 30$ . Also,  $y = 4x = 120$ .