Assam Academy of Mathematics

MATHEMATICS OLYMPIAD

September 9, 2018 (Sunday)

Category-III ***(for Classes IX, X & XI)***

(Figures in the margin indicate full marks for the questions.)

Q. 1. If a, b, c are positive real numbers, prove that

$$3\left(a+\sqrt{ab}+\sqrt[3]{abc}\right)\le 4(a+b+c)$$ 5

Solution : using $AM \geq GM -$

$$\sqrt{ab} = \frac{1}{2}\sqrt{4ab} = \frac{1}{2}\sqrt{a.4b} \leq \frac{1}{2}\times\frac{a+4b}{2}$$

$$\sqrt[3]{abc} = \frac{1}{4}\sqrt{16abc} = \frac{1}{4}\sqrt{a.4b.4c} \leq \frac{1}{4}\times\frac{a+4b+4c}{3}$$

Thus, $3\left(a+\sqrt{ab}+\sqrt[3]{abc}\right)$

$$\leq 3\left[a+\frac{a+4b}{4}+\frac{a+4b+4c}{12}\right]$$

$$=\frac{3}{12}\left[12a+3a+12b+a+4b+4c\right]$$

$$=\frac{1}{4}\left[16a+16b+4c\right]$$

$$\leq\frac{1}{4}\left[16a+16b+16c\right]=4(a+b+c)$$

Q. 2. The number N = 999 ..... 9 consists of exactly 2018 nines. Compute the number N^3. How many nines are there in N^3 ? 6

Solution : Let N = 999 ....9 (n nines)

$\Rightarrow N = 10^n - 1$

$\Rightarrow N^3 = (10^n - 1)^3$

$= 10^{3n} - 3 \times 10^{2n} + 3 \times 10^n - 1$

$= (10^n - 3) \times 10^{2n} + 2 \times 10^n + 10^n - 1$

$$= \underbrace{999\ \ldots\ldots\ 9}_{(n-1)\text{ nines}}7 \times 10^{2n} + [2 \times 10^{n} + \underbrace{999\ \ldots\ 9}_{n\text{ nines}}]$$

$$= \underbrace{999\ \ldots\ldots\ 9}_{(n-1)\text{ nines}}7\ \underbrace{000\ \ldots\ldots\ 0}_{2n\text{ zeros}} + 2\,\underbrace{999\ \ldots\ldots\ 9}_{n\text{ nines}}$$

$$= \underbrace{999\ \ldots\ldots\ 9}_{(n-1)\text{ nines}}7\ \underbrace{000\ \ldots\ldots\ 0}_{(n-1)\text{ zeros}}\ 2\ \underbrace{99\ \ldots\ldots\ 9}_{n\text{ nines}}$$

Clearly the no. of nines in N^3 is $(2n - 1)$.

Q. 3. How many numbers less than 1000 have the property that the sum of the digits is divisible by 7 and the number ifself is divisible by 3?7

Solution : Let n be one such number. By question, $3|n$. Then $3|s(n)$ also, s(n) being the sum of digits of n. Also $7|s(n)$. But ged $(3,7) = 1$

Hence $3 \times 7 = 21|s(n)$

Now, the number is less than 1000 and so it is of at most three digits. Maximum sum of the digits is 27. Since $21|s(n)$, so the only possibility is $s(n) = 21$. So, n cannot be of one or two digits. In other words, n has to be a 3 digit number.

Let $n = xyz$, so $x+y+z = 21$

x	y	z	No. of numbers
3	9	9	$\frac{\lfloor 3}{\lfloor 2} = 3$
4	8	9	$\lfloor 3 = 6$
5	7	9	$\lfloor 3 = 6$
5	8	8	$\frac{\lfloor 3}{\lfloor 2} = 3$
6	7	8	$\lfloor 3 = 6$
6	6	9	$\frac{\lfloor 3}{\lfloor 2} = 3$
7	7	7	1

Total number of 3 digit numbers satistying the given conditions is 28.

Q. 4. Consider the quadratic equation $ax^2 + bx + c = 0$, where a, b, c are odd integers. Show that this equation cannot have rational roots. 7

Solution : If possible, let the equation have rational roots.

Let $\alpha = \frac{p}{q}$ be a rational root with p, q as odd integers and

gcd (p, q) = 1 (2)

Then, $a\left(\frac{p}{q}\right)^2 + b\left(\frac{p}{q}\right) + c = 0$

$\Rightarrow ap^2 + bpq + cq^2 = 0$

$\Rightarrow -ap^2 = q(bp + cq)$

Now gcd (p, q) = 1 $\Rightarrow$ $q \mid a$ since $q \mid p^2$

But a is odd and therefore q is also odd.

Similarly, $-cq^2 = ap^2 + bpq = p\,(ap + bq)$

Since $p \mid q^2$ therefore $p \mid c$

Now c being odd, p is also odd.

Thus $ap^2 + bpq + cq^2$ is an odd number which means

$ap^2 + bpq + cq^2 \neq 0$

$\therefore \alpha = \frac{p}{q}$ cannot be a root of the equation.

Q. 5. How many integers greater than 5400 have both of the following properties :

(a) the digits are distinct.

(b) the digits 2 and 7 do not occur. 7+3=10

Solution : There are 10 distinct digits from 0 to 9. Excluding 2 and 7, there are 8 distinct digits.

Let us first consider 5 digit numbers.

Total no. of 5 digit numbers is 7 × 7 × 6 × 5 × 4

Number of 6 digit numbers is 7 × 7 × 6 × 5 × 4 × 3

Number of 7 digit numbers is 7 × 7 × 6 × 5 × 4 × 3 × 2

Number of 8 digit numbers is 7 × 7 × 6 × 5 × 4 × 3 × 2 × 1

Now let us consider 4 digit numbers greater than 6000.

No. of such numbers is 4 × 7 × 6 × 5

No. of 4 digit numbers greater than 5400 and less than 6000 is 4 × 6 × 5

Thus total number of numbers satisfying the two conditions is

7 × 7 × 6 × 5 × 4 + 7 × 7 × 6 × 5 × 4 × 3 + 7 × 7 × 6 × 5 × 4 × 3 × 2 + 7 × 7 ×

$6 \times 5 \times 4 \times 3 \times 2 \times 1 + 4 \times 7 \times 6 \times 5 + 4 \times 6 \times 5$

$= 7 \times 7 \times 6 \times 5 \times 4\ (1 + 3 + 6 + 6) + 4 \times 6 \times 5\ (7 + 1)$

$= 49 \times 30 \times 4 \times 16 + 120 \times 8$

$= 1470 \times 64 + 960$

$= 95040$

Q. 6. Find the sum of the series :

$1^2 - 2^2 + 3^2 - 4^2 + ... + (-1)^{n+1}\ n^2.$ 8

Solution : The given series

$= (1^2 + 3^2 + 5^2 + ... + n^2) - \{2^2 + 4^2 + ... + (n-1)^2\}$

$= (1^2 + 2^2 + 3^2 + ... + n^2) - 2\{2^2 + 4^2 + ... + (n-1)^2\}$

$$= \frac{n(n+1)\ (2n+1)}{6} - 2 \times 2^2\left\{1^2 + 2^2 + ... + \left(\frac{n-1}{2}\right)^2\right\}$$

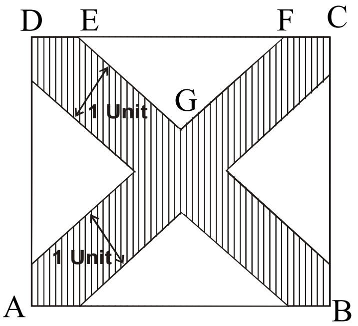
$$= \frac{n(n+1)\ (2n+1)}{6} - 8\,\frac{\frac{n-1}{2}\left(\frac{n-1}{2}+1\right)\left\{2\,\frac{n-1}{2}+1\right\}}{6}$$

$$= \frac{n(n+1)\ (2n+1)}{6} - \frac{(n-1)\ (n+1)n}{3}$$

$$= \frac{n(n+1)}{6}\left[2n+1-2n+2\right]$$

$$= \frac{n(n+1)}{2}$$

Q. 7. The diagonals of a square tile are painted symmetrically with a brush of width I unit, as shown in figure. Exactly half of the area of this tile is covered with paint. What is the length of the side of the tile? 8



Solution : Let DE = x, EG = y

Clearly, $1^2 = x^2 + x^2 = 2x^2 \Rightarrow x = \frac{1}{\sqrt{2}}$

$EF = CD - 2DE$

$= CD - \frac{2}{\sqrt{2}} = CD - \sqrt{2}$

Now, $EF^2 = EG^2 + GF^2 = y^2 + y^2$

$\therefore \quad 2y^2 = (CD - \sqrt{2})^2$

$\Rightarrow CD - \sqrt{2} = \sqrt{2}y$

According to question–

$$\frac{1}{2}CD^2 = 4.\left(\frac{1}{2}y^2\right)$$

$$\Rightarrow CD = 2y = \sqrt{2}\left(CD - \sqrt{2}\right)$$

$$\Rightarrow \left(\sqrt{2} - 1\right)CD = 2$$

$$\Rightarrow CD = \frac{2}{\sqrt{2} - 1} = 2\left(\sqrt{2} + 1\right)$$

Q. 8. Let f(x) be a monic polynomial (of degree 10) say

$f(x) = x^{10} + a_1 x^9 + a_2 x^8 + ... + a_9$

where the cofficients a's are integers. If all the roots of f(x) = 0 are from the set {1, 2, 3}, find the number of such polynomials, If g(x) is the sum of all such polynomials, show that the constant term of g(x) is $\frac{1}{2}\left(3^{12} + 1\right) - 2^{12}$.

3+7=10

Solution : Since the polynomial is monic and its roots are from the set {1, 2, 3}, so it can be written as

$f(x) = (x-1)^p (x-2)^q (x-3)^r$, where $o \leq p, q, r \leq 10$ and $p + q + r = 10$

No. of such polynomials can be found by finding the number of triple (p, q, r) of non-negative integers satisfying p + q +r = 10. Let us make the table–

p	r = 10 – q – p	q can take values	No. of triples
0	r = 10 – q	0, 1, 2, ..., 10	11
1	r = 9 – q	0, 1, 2, ..., 9	10

2	r = 8 − q	0, 1, ..., 8	9
3	r = 7 − q	0, 1, ..., 7	8
4	r = 6 − q	0, 1, ..., 6	7
5	r = 5 − q	0, 1, ..., 5	6
6	r = 4 − q	0, 1, ..., 4	5
7	r = 3 − q	0, 1, ..., 3	4
8	r = 2 − q	0, 1, 2	3
9	r = 1 − q	0, 1,	2
10	r = − q	0	1

Hence total number polynomials is 1 + 2 + 3 + ... 11

$$= \frac{11\times 12}{2} = 66$$

Now $g(x) = \sum_{p,q,r} (x-1)^p (x-2)^q (x-3)^r$

Contant term of $g(x)$ is

$$\sum_{p+q+r=10} (-1)^p (-2)^q (-3)^r$$

$$= \sum_{p+q+r=10} (-1)^{p+q+r} 1^p 2^q 3^r$$

$$= \sum_{p+q+r=10} 1^p 2^q 3^r$$

$$= 1^0 \left(2^0\times 3^{10} + 2^1\times 3^9 + 2^2\times 3^8 + 2^3\times 3^7 + 2^4\times 3^6 + 2^5\times 3^5 \ldots + 2^{10}\times 3^0\right)$$

$$+1^1 \left(2^0\times 3^9 + 2^1\times 3^8 + 2^2\times 3^7 + \ldots + 2^9\times 3^0\right)$$

$$+1^1 \left(2^0\times 3^8 + 2^1\times 3^7 + 2^2\times 3^6 + \ldots + 2^8\times 3^0\right)$$

$$+1^3 \left(2^0\times 3^7 + 2^1\times 3^6 + 2^2\times 3^5 + \ldots 2^7\times 3^0\right)$$

$$+1^4 \left(2^0\times 3^6 + 2^1\times 3^5 + 2^2\times 3^4 + 2^3\times 3^3 + 2^4\times 3^2 + 2^5\times 3\times 2^6\times 3^0\right)$$

$$+1^5 \left(2^0\times 3^5 + 2^1\times 3^4 + 2^2\times 3^3 + 2^3\times 3^2 + 2^4\times 3^1 + 2^5\times 3^0\right)$$

$$+1^6 \left(2^0\times 3^4 + 2^1\times 3^3 + 2^2\times 3^2 + 2^3\times 3^1 + 2^4\times 3^0\right)$$

$$+1^8 \left(2^0\times 3^2 + 2^1\times 3^1 + 2^2\times 3^0\right)$$

$$+1^9\left(2^0\times3^1+2^1\times3^0\right)$$

$$+1^{10}\left(2^0\times3^0\right)$$

$$=2^0\left(1+3+3^2+\ldots+3^{10}\right)+2^1\left(1+3+3^2+\ldots3^9\right)$$

$$+2^2\left(1+3+3^2+\ldots+3^8\right)+2^3\left(1+3+\ldots+3^7\right)$$

$$+2^4\left(1+3+\ldots+3^6\right)+2^5\left(1+3+\ldots+3^5\right)+2^6\left(1+3+\ldots3^4\right)$$

$$+2^7\left(1+3+3^2\ldots+3^3\right)+2^8\left(1+3+3^2\right)+2^9\left(1+3\right)$$

$$+2^{10}\left(1+3^0\right)$$

$$=\frac{3^{11}-1}{3-1}+2^1\times\frac{3^{10}-1}{3-1}+2^2\times\frac{3^9-1}{3-1}+2^3\times\frac{3^8-1}{3-1}+2^4\times\frac{3^7-1}{3-1}$$

$$+2^5\times\frac{3^6-1}{3-1}+2^6\times\frac{3^5-1}{3-1}+2^7\times\frac{3^4-1}{3-1}+2^8\times\frac{3^3-1}{3-1}+2^4\times\frac{3^7-1}{3-1}$$

$$+2^9\times\left(\frac{3^2-1}{3-1}\right)+2^{10}\times\left(\frac{3-1}{3-1}\right)$$

$$=\frac{3^{11}-1}{2}+2^0\times3^{10}+2\times3^9+2^2\times3^8+2^3+2^3\times3^7+2^4\times3^6+2^5\times3^5$$

$$+2^6\times3^4+2^7\times3^3+2^8\times3^2+2^9\times3+2^{10}\times3^0$$

$$-\left(1+2+2^2+\ldots+2^9\right)-2^{10}\times3^0$$

Q. 9. In a room there are 10 people, none of whom are older than 60 years (ages are given in whole numbers only) but each of whom is at least 1 year old. Prove that one can always find two groups of people (with no common person) such that the sum of ages of the people in each group is the same.

Soln. No. of non empty subsets of 10 people in the room is 2^{10}-1=1024-1=1023 Clearly, the sum of ages of people in these 1023 subsets varies from 1 to 10 × 60=600. In other words, there are 1023 non empty subsets of 10 people and the sum of ages of each of these subsets is a number lying between 1 and 600. Numbering 600 boxes as 1, 2, 3, ...600, the 1023 subsets can be distributed into these boxes and since 1023>600, by Pigeon Hole Principle (PHP), at least one box will contain more than one subset. Let A and B be two such subsets belonging to the same box.

Then, the sum of ages of people in A = the sum of ages of people in B.

If $A \cap B = \phi$ then the problem is solved.

If $A \cap B \neq \phi$, then some people will be common to both A and B.

Let $\quad A \cap B = \{P_1, P_2 ...., P_n\}$

Define $C = A - \{P_1, P_2 ......P_n\}$

$$D = B - \{P_1, P_2 .....P_n\}$$

Since $A \neq B$, therefore $C \neq D$ and $C \cap D = \phi$

Also sum of ages of people in C = Sum of ages of people in D.

Hence proved.

Q10. Find the function $f : N \to N$ such that

(a) $f(n)$ is a perfect square for each $n \in N$

(b) $f(m+n) = f(m) + f(n) + 2mn, \quad m, n \in N.$

Soln. By (a), let us assume that

$$f(1) = \alpha^2, \quad \alpha \in N$$

$$\therefore f(2) = f(1+1) = f(1) + f(1) + 2 \times 1 \times 1$$
$$= \alpha^2 + \alpha^2 + 2$$
$$= 2(\alpha^2 + 1)$$

$$f(3) = f(2) + f(1) + 2 \times 2 \times 1$$
$$= 2(\alpha^2 + 1) + \alpha^2 + 4$$
$$= 3\alpha^2 + 6$$
$$= 3(\alpha^2 + 2)$$

$$f(4) = f(3+1) = f(3) + f(1) + 2 \times 3 \times 1$$
$$= 3(\alpha^2 + 2) + \alpha^2 + 6$$
$$= 4\alpha^2 + 12$$
$$= 4(\alpha^2 + 3)$$

—

So, Let $f(k) = k(\alpha^2 + k - 1)$

Now $f(k+1) = f(k) + f(1) + 2k \times 1$

$$= k(\alpha^2 + k - 1) + \alpha^2 + 2k$$

$$= (k+1)\alpha^2 + k^2 + k$$

$$= (k+1)[\alpha^2 + k]$$

$$= (k+1)[\alpha^2 + (k+1) - 1]$$

By induction $f(n) = n(\alpha^2 + n - 1) \quad \forall n \in N$

But $f(n)$ is a perfect square

For any prime p

$f(p) = p(\alpha^2 + p - 1)$ which is also a perfect square. So $p^2 m$ not be a factor of $f(p)$. This means,

$$p \mid \alpha^2 + p - 1$$

$$\Rightarrow p \mid (\alpha^2 + p - 1) - p \qquad \text{since } p \mid p$$

$\Rightarrow p \mid \alpha^2 - 1$ for any prime p

i.e, $\alpha^2 - 1$ is divisible by all primes

This is possible if and only if $\alpha^2 - 1 = 0$

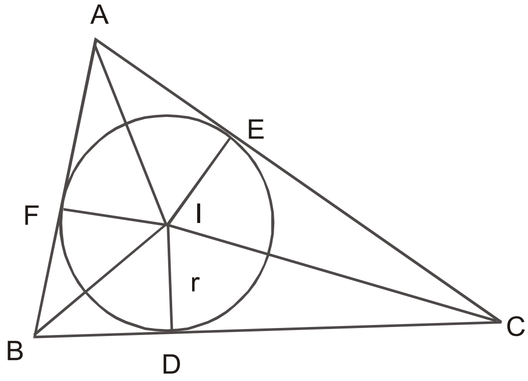
$$\Rightarrow \alpha = 1$$

$\therefore$ f(x) = n $(1^2 + n - 1) = n^2 \qquad \forall n \in N$

Q. 11. In a triangle ABC, show that the inradius is given by $r = \frac{\Delta}{s}$ where Δ is the area of the triangle and s is the semi-perimeter of the triangle. Also, show that

$$r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

where R is the circumradius of the triangle. 5+2=7



Solution : Let r be the inradius of ΔABC with I as incentre.

Area ΔABC = Area of ΔIBC + Area of ΔICA + Area of ΔIAB.

$$= \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$$

$$= \frac{1}{2}(a+b+c)r$$

$= sr$ where s = semiperimeter

$$= \frac{a+b+c}{2}$$

Next, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$\therefore$ a = 2R sin A, b = 2R sin B, C = 2R sin C

Also from Δ BID, $BD = ID\cot\frac{B}{2} = r\cot\frac{B}{2}$

and from ΔCID, CD $= r\cot\frac{C}{2}$

$\therefore$ a = BC = BD+CD = r (cot $\frac{B}{2}$ + cot $\frac{C}{2}$)

or 2R sin A = r $\dfrac{\sin\frac{C}{2}\cos\frac{B}{2} + \cos\frac{C}{2}\sin\frac{B}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}$

$$= r \frac{\sin\frac{B+C}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$i.e., \ 4R \sin\frac{A}{2}\cos\frac{A}{2} = \frac{r\sin\left(\frac{\Pi}{2}-\frac{A}{2}\right)}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$\Rightarrow r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

Q. 12. Consider the set {1, 2, 3, ..., n}. A derrangement of S_n is a permulation i_1 i_2 i_3 .... i_n of 1, 2, 3, ..., n. such that $i_j \neq j \ \forall \ j = 1, 2, 3, ..., n$. In other words, a derangement of {1, 2, 3, ..., n} is a permutation of its elements in which no integer is in its natural position. For example, 231 is derangement of {1, 2, 3}. Let D_n denote the number of derangements of {1, 2, 3, ..., n}. Find D_1, D_2, D_3 and D_4 by actually listing the dearrangements. Next, use the principle of inclusion and exclusion to express D_n in the form.

$$D_n = n\left(1-\frac{1}{1}+\frac{1}{2}-\frac{1}{3}+....+(-1)^n\frac{1}{n}\right)$$

(you may work with n = 2, 3, 4 only 2+2+3+5

Solution : $D_1 = 0$

$D_2 = 1$

$D_3 = 2$ $\quad \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$

$D_4 = 9$

$$\left\{\begin{matrix}\begin{pmatrix}1234\\2341\end{pmatrix}\begin{pmatrix}1234\\3412\end{pmatrix}\begin{pmatrix}1234\\4123\end{pmatrix}\\ \begin{pmatrix}1234\\2143\end{pmatrix}\begin{pmatrix}1234\\3412\end{pmatrix}\begin{pmatrix}1234\\4321\end{pmatrix}\\ \begin{pmatrix}1234\\2413\end{pmatrix}\begin{pmatrix}1234\\3142\end{pmatrix}\begin{pmatrix}1234\\4213\end{pmatrix}\end{matrix}\right.$$

Now consider n = 2, The total number of permutations of {1, 2} is 2 = 2

Let A_1 = Set of permutations where 1 is in its original position.

Then $n(A_1) = 1$

Let A_2 = set of permutations where 2 is in its original position $n(A_2) = 1$

$\therefore A_1 \cap A_2$ = Set of permutations where both 1 and 2 are in original position. This happens only in 1 case.

$n(A_1 \cap A_2) = 1$

No. of dearrangements

$= n\left(A_1^c \cap A_2^c\right)$

= no. of permutations where neither 1 nor 2 is in its original position.

$= n(A_1 \cup A_2)^c$

$= n(U) - n(A_1 U A_2)$, U is the set of all permutations

$= \underline{|2} - [n(A_1) + n(A_2) - n(A_1 \cap A_2)]$

$= \underline{|2} - [1 + 1 - 1]$

$= \underline{|2} - [2 - 1] = \underline{|2} - \underline{|2} + 1 = \underline{|2}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{2}\right)$

For n = 3, let A_1 = Set of permulations of order i where i is in its original position. i = 1, 2, 3

$\therefore n(A_1) = 2$

Also $n(U) = 3$

$n(A_1 \cap A_2) = n(A_2 \cap A_3) = n(A_1 \cap A_3) = 1$

and $n(A_1 \cap A_2 \cap A_3) = 1$

$\therefore$ No. of dearrangemts

$= n\left(A_1^c \cap A_2^c \cap A_3^c\right)$

$= n(A_1 \cup A_2 \cup A_3)^c$

$= n(U) - n(A_1 \cup A_2 \cup A_3)$

$= n(U) - [n(A_1) + n(A_2) + n(A_3) - n(A_1 \cap A_2)$

$- n(A_1 \cap A_3) - n(A_2 \cap A_3) + (A_1 \cap A_2 \cap A_3)]$

$= 3 - [\underline{|2} + \underline{|2} + \underline{|2} - \underline{|1} - \underline{|1} - \underline{|1} + 1]$

$= \underline{|3} - 3\underline{|2} + 3\underline{|1} - 1$

$$= \underline{|3}\left[1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}}\right]$$

For n = 4, $n(A_i) = 3$ (i th place is fixed, remaining 3 places can be filled in 3 ways.)

$n(A_i \cap A_j) = 2 \quad (i \neq j)$

$n(A_i \cap A_j \cap A_k) = 1 \quad i \neq j,\ j \neq k,\ i \neq k$

$n(A_i \cap A_2 \cap A_3 \cap A_4) = 1$

$n(\cup) = 4$

No. of dearragements

$$= n\left(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c\right)$$

$$= n(\cup) - [n(A_1) + n(A_2) + n(A_3) + n(A_4) - n(A_1 \cap A_2)$$

$$- n(A_1 \cap A_3) - n(A_1 \cap A_4) - n(A_2 \cap A_3) - n(A_2 \cap A_4)$$

$$- n(A_3 \cap A_4) + n(A_1 \cap A_2 \cap A_3) + n(A_1 \cap A_2 \cap A_4) +$$

$$(A_1 \cap A_3 \cap A_4) + n(A_2 \cap A_3 \cap A_4) - n(A_1 \cap A_2 \cap A_3 \cap A_4)]$$

$= 4 - [4\ 3 - 6\ 2 + 4\ 1 - 1]$

$$= \underline{|4}\left[1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \frac{1}{4}\right]$$

Thus no. ofderrangements of n is $D_n = \frac{1}{n}\left[1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{3} + \ldots\ldots\ldots\ldots + (-1)^n \frac{1}{n}\right]$