

## Assam Academy of Mathematics

### MATHEMATICS OLYMPIAD

September 9, 2018 (Sunday)

Category-I (for Classes V & VI)

*(Figures in the margin indicate full marks for the questions.)*

**Q. 1.**

4×5=20

(a) Fill in the blank

1, 2, ?, 15, 31, 56, .....

Sol. Writing  $r^{\text{th}}$  term =  $t_r$ , we note by inspection

$$t_r = t_{r-1} + (r-1)^2$$

Since  $t_2 = 2 = 1 + 1 = t_1 + 1^2$

$$t_5 = 31 = 15 + 16 = t_4 + 4^2$$

$$t_6 = 56 = 31 + 25 = t_5 + 5^2$$

Thus  $t_3 = t_2 + 2^2 = 2 + 2^2 = 6$

∴ Ans 6

(b) If BOY is coded as YLB then uncode NZM.

Sol. The letters of BOY and YLB are symmetrically positioned.

Therefore, symmetrically opposite arrangement of NZM will be MAN.

Ans. MAN

(c) In a group of goats and ducks the total number of legs is 20 more than twice the number of their heads. What is the number of goats?

Sol. If  $m$  and  $n$  be the number of goats and ducks

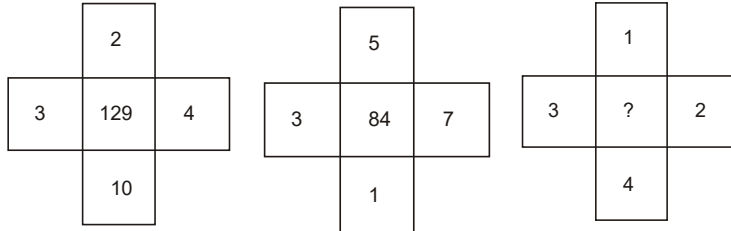
$$\text{then } 2(m+n) + 20 = 4m + 2n$$

i.e.,  $2m = 20$

So,  $m = 10$

Ans. Number of goats is 10 (whatever be the number of ducks).

(d) Fill in the missing number



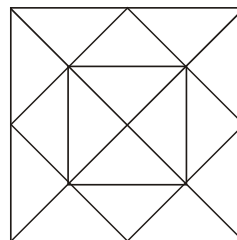
Sol. In the 1st pattern,  $2^2 + 3^2 + 10^2 + 4^2 = 129$

In the 2nd pattern,  $1^2 + 3^2 + 5^2 + 7^2 = 84$

Therefore in the 3rd pattern the missing figure is  $1^2 + 2^2 + 4^2 + 3^2 = 30$

Ans. 30

(e) Count the number of triangles in the figure



Sol. Ans. 32

[Award partial marks depending upon ability to count 12 or more than 12 triangles]

**Q. 2.**

10

A, B, C, D and E are five students of class VI with 11 years as their average age. The two groups each consisting of students A, B, D, E and A, C, D, E respectively also have 11 years as their average ages. E is older than both A and D by 3 years. What are the individual ages of the students?

Sol. Let A, B, C, D, E denote both the students and their respective ages.

$$\text{Then } A + B + C + D + E = 11 \times 5 = 55 \text{ — (i)}$$

$$A + B + D + E = 11 \times 4 = 44 \text{ — (ii)}$$

$$A + C + D + E = 11 \times 4 = 44 \text{ — (iii)}$$

$$\text{(i) - (ii)} \Rightarrow C = 10$$

$$\text{(i) - (iii)} \Rightarrow B = 10$$

By (i) again  $A + D + E = 55 - 20 = 35$

Since E is older than both A and D by 3 years, therefore  $A = D$  and

$$E = A + 3 = D + 3$$

Thus  $A + A + A + 3 = 35$

$$\Rightarrow 3A = 32$$

$$\therefore A = \frac{32}{3} = D \text{ and}$$

$$E = \frac{32}{3} + 3 = \frac{41}{3}$$

$$\text{Ans. } A = \frac{32}{3}, B = 10, C = 10, D = \frac{32}{3}, E = \frac{41}{3}$$

**Q. 3.**

10

A bus started from a bus station with some passengers. In the next stop, half of the passengers got down while 18 others boarded the bus. In the second stop one third of the passengers again got down while 16 others boarded the bus. The bus then moved towards its destination with the same number of passengers initially boarding the bus. How many passengers initially boarded the bus?

Soln. Let  $x$  be the number of passengers boarding initially

At the next stop the bus had  $\frac{x}{2} + 18$  passengers

At the second stop the bus had

$$\frac{2}{3} \left( \frac{x}{2} + 18 \right) + 16$$

By condition

$$\frac{2}{3} \left( \frac{x}{2} + 18 \right) + 16 = x$$

$$\Rightarrow \frac{2}{3}x = 28$$

$$\Rightarrow x = 42$$

**Q. 4.**

$5 \times 2 = 10$

When 325 and 535 are divided by a certain number there is left the same remainder

10 in each case. Find the number.

Soln.  $325 - 10 = 315 = 3 \times 105 = 3 \times 3 \times 35 = 3 \times 3 \times 5 \times 7$

$535 - 10 = 525 = 5 \times 105 = 3 \times 5 \times 35 = 3 \times 5 \times 5 \times 7$

HCF (315, 525) =  $3 \times 5 \times 7 = 105$

Ans. 105

**Q. 5.** 10

Find the greatest number that will divide 705, 1805 and 1475 leaving in each case the same remainder.

Soln. Let  $m$  be the greatest number which leaves remainder  $r$  on dividing 705, 1805 and 1475.

Then  $705 = mq_1 + r$  — (i)

$1805 = mq_2 + r$  — (ii)

$1475 = mq_3 + r$  — (iii)

Then (ii) - (i)  $\Rightarrow m(q_2 - q_1) = 1805 - 705 = 1100$

(ii) - (iii)  $\Rightarrow m(q_2 - q_3) = 1805 - 1475 = 330$

Thus  $m$  is a common factor of 330 and 1100

Now  $330 = 11 \times 30 = 11 \times 3 \times 10$

$1100 = 11 \times 100 = 11 \times 10 \times 10$

Hence the greatest common factor of 330 and 1100 is  $11 \times 10 = 110$

i.e.  $m = 110$

**Q. 6.** 10

Of a certain dynasty  $\frac{1}{3}$  of the kings were of the same name,  $\frac{1}{4}$  of another,  $\frac{1}{8}$  of

another,  $\frac{1}{12}$  of still another name, and there were 5 besides. How many kings were there of each name?

Soln. Let  $x$  be the total number of Kings in the dynasty. Then

$$x = \frac{x}{3} + \frac{x}{4} + \frac{x}{8} + \frac{x}{12} + 5$$

$$\Rightarrow x \left( 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{8} - \frac{1}{12} \right) = 5$$

$$\Rightarrow x \times \frac{5}{24} = 5$$

$$\Rightarrow x = 24$$

$\therefore$  No. of Kings of each name are

$$\frac{24}{3} = 8, \quad \frac{24}{4} = 6, \quad \frac{24}{8} = 3, \quad \frac{24}{12} = 2$$

and 1 each with 5 different names.

**Q. 7.** 10

The cost of carpeting a room is Rs. 375. Had the length of the room been shorter by 3 meters, the cost would have been Rs. 330. Find the length of the room.

Soln. Let  $l$  and  $b$  be the length and breadth of the room. Let the cost of carpeting per square metre be Rs.  $x$ .

$$\therefore lbx = 375, \quad (l-3)bx = 330$$

$$\therefore \frac{lbx}{(l-3)bx} = \frac{375}{330} = \frac{25}{22}$$

$$\therefore 22l = 25(l-3) \quad \Rightarrow l = 25 \text{ meters}$$

**Q. 8.** 10

A boy had to divide 7865321 by 254. He copied a figure wrongly in the divisor and obtained 33612 as quotient with remainder 113. What mistake did he make?

Soln. Here, after division of 7865321 the boy obtained 33612 as quotient and 113 as the remainder.

$$\begin{aligned} \text{Hence the divisor is } & (7865321 - 113) \div 33612 \\ & = 7865208 \div 33612 \\ & = 234 \end{aligned}$$

So, he made the mistake in writing 5.

In other words he wrote 234 instead of 254

**Q. 9.** 10

Study the following pattern and fill up the blank.

8	14	7	4
22	(?)	5	11
26	13	6	12

Soln. 1st row:  $8 \times 7 = 14 \times 4 = 56$

3rd row:  $26 \times 6 = 13 \times 12 = 156$

$\therefore$  2nd row:  $22 \times 5 = (?) \times 11$

$$\therefore (?) = \frac{22 \times 5}{11} = 10$$

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