Fibonacci Sequence and The Plant Kingdom

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Abstract. The Fibonacci sequence is an amazing sequence followed by the nature itself. This may be treated as fundamental sequence because the nature and the natural phenomena follow the theory. The Golden Ratio is applicable to geometry, structure and anatomy of human, drawing of paintings, creation of architectural design and even in the field of financial activity, etc.

The aim of this work is to show how the Golden Ratio is applicable to the pattern of tree leaves and relation of Fibonacci sequence with the design of flower petals, exist in the nature. Here, we mainly focus on the pattern of flower petals and number of petals which are commonly available in our surroundings. The natural phenomenon includes the symmetries of tree leaves, flower petal patterns, human anatomy even waves of sea water, etc.

Natural patterns include symmetries, trees, spirals, meanders, waves, foams, tessellations, crack and stripes. The study performed by me is also related to how human anatomy is related to the Fibonacci sequence.

In this work, it is shown that the ratio of length and average breadth of tree leaves follow the golden ratio. We have also demonstrated through this work that the number of petals of flowers collected locally is in the order of Fibonacci sequence.

1. Introduction

As we explore our surroundings in nature more and more, we see that several facts and observations can be described mathematically. A strong correlation is observed between nature and mathematics. Most of us are not interested in going deep about what mathematical explanations are in nature. According to the famous mathematician Euclid, “The laws of nature are but the mathematical thoughts of God”. Mathematics is all around us. Mathematical formulations and justifications are inherent in our universe; in the cases of plants, animals, geographical observations, fractals, shapes and patterns and, many more.

There are many examples in the world of plants. A few examples include the number of spirals in a pine cone, pineapple or seeds in a sunflower, the number of petals and symmetry on a flower. The concentric circles are observed in the layers of an onion and the rings of trees that form as it grows and ages.

Mathematical symmetry is inherent everywhere in nature. The human body would be an excellent example of a living being that has symmetry. A dihedral 5 symmetry is observed in starfish's body structure with five rotations of 72° each and have five lines of reflection. Hexagons are six-sided polygons that fit most closely together without any gaps. The hexagonal structure of wax cells that bees create to store their eggs and larvae is an excellent example of mathematical optimisation of space.

Many birds follow their flight path using celestial objects like the Sun or stars and unconscious trigonometric calculations to navigate their flight. Birds solve the critical trigonometry problems in the same way a waterfall follows the Navier-Stokes equation or a dog finds an object.

Natural objects have some definite shapes. Geometry is the branch of maths that describes the shapes of many natural objects. For example, cones are formed by volcanoes. The steepness and height of the cones depend on the runniness (viscosity) of the lava. Fast, runny lava forms flatter cones, but the thick viscous lava forms steep-sided cones. Cones are 3-dimensional solids whose volume can be calculated as ($1/3 \times$ area of base $\times$ height).

The building blocks of the natural world can easily be explained in terms of some Mathematical formulations. A fractal is a never-ending recursive pattern recursive where the same basic shape is repeated again and again in shape itself. Fractals are another intriguing mathematical structure that is observed in nature. Fractals make up many aspects of our world, including the leaves of ferns, the branches of the trees, the branching of neurons in our brains, and coastlines.

Mathematics helps to interpret all the activities of our brain. When information is being processed governed by a set of patterns like those found in the branches of a tree or the fluid dynamics of a waterfall the way it feels is consciousness. Basically, consciousness is a mathematical pattern. Here are a few of my favourite examples of math in nature, but many other stones are left unturned.

We can understand the order of the universe by studying the patterns that emerge from it. Even something what we see randomly as the shape of a tree's branches has an order. The main trunk of a tree will grow until it produces a branch that has two growth points and each stem branches into two. Interestingly, this pattern is repeated for every new stem.

The Fibonacci sequence is a unique mathematical sequence followed by nature itself. Fibonacci sequence was proposed by the famous mathematician Fibonacci Leonardo Pisano is popularly known as Fibonacci. This Sequence is a peculiar series of numbers from classical mathematics that has found applications in advanced mathematics, nature, statistics, computer science, etc. It may be treated as fundamental sequence because this sequence can explain several natural phenomena. Surprisingly, it is observed that numerous natural facts follow this sequence and can be thought of as the best secret code of nature. It can well explicate the dimensions of the Pyramid of Giza, the vast sea shells, famous paintings, famous architectural designs and many more.

The Fibonacci sequence is a series of numbers starting with 0 and 1 and the sum of the two preceding numbers form the next number. The mathematical rule to find any Fibonacci number (F) of the sequence is $F_n = F_{n-1} + F_{n-2}$, where $F_1 = 0, F_2 = 1, n \geq 3$. The sequence is then given by $0, 1, 1, 2, 3, 5, 8, 13, \ldots$.

The average ratio of every two consecutive terms in the Fibonacci sequence gives us a famous constant, popularly known as the golden ratio and denoted by φ, where $\varphi = 1.618033989\ldots$. We can find examples of the golden ratio everywhere in nature. This aesthetically appealing ratio is found in geometrical shapes, plant structures, drawings and paintings, architectural designs, and even in the field of financial activities. There are many other examples in our surroundings to be explored. We can find examples of the golden ratio everywhere in nature.

This work is an initiative in this direction. This work aims to search some new natural examples in our surroundings where golden ratio is applicable. As a part of empirical studies, two case examples regarding the connotation of the Fibonacci sequence and Golden Ratio with the pattern of flower petals and leaf structure of some commonly available plants in the different areas of Assam, India, are reflected below.

2. Empirical Studies

2.1. Case example 1: Fibonacci sequence and the pattern of flower petals

In nature, the flowers of different species have some definite patterns and number of petals. An exciting relation is observed between the Fibonacci sequence and the structure of petals of some common flowers. The number of petals is equal to a term of the Fibonacci sequence: $0, 1, 1, 3, 5, 8, 13, \ldots$. For the present study, several common flowers and their photograph have been collected from the said locality. The photographs of 16 arbitrarily chosen flowers are presented below.

Table 1 (below): Represents the common names, scientific names and the number of petals of the 16 flowers. The observations are presented in the last column.

Sl No.	Local Names	Scientific Names	Number of petals	Observation
1.	Kori flower	*Tabernaemontana*	5	Fibonacci Number
2.	Zinnia	*Zinnia elegans*	13	Fibonacci Number
3.	Jaba	*Hibiscus rosa-sinensis*	5	Fibonacci Number
4.	Tara gandha	*Tagetes erecta Linn*	8	Fibonacci Number
5.	Aparajita/Asian pigeonwings	*Clitoria terntea*	2	Fibonacci Number
6.	Korobi/Oleander	*Nerium oleander*	5	Fibonacci Number
7.	Halud flower	*Unknown*	5	Fibonacci Number
8.	Wild flower	*Unknown*	5	Fibonacci Number
9.	Kachu /Arum flower	*Colocasia esculenta*	1	Fibonacci Number
10.	Bokful /Heron flower	*Sesbania grandiflora*	5	Fibonacci Number
11.	Bougainvillea	*Bougainvillea glabra*	3	Fibonacci Number
12.	Kanchan	*Bauninia acuminata*	5	Fibonacci Number
13.	Rose	*Rosa*	5	Fibonacci Number
14.	Sapla	*Nymphaeaceae*	8	Fibonacci Number
15.	Nayantara	*Catharanthus roseus*	5	Fibonacci Number
16.	White water lily	*Nymphaea alba*	8	Fibonacci Number

2.2. Case example 2: Golden Ratio and the structure of tree leaves

The trees of different species have leaves with different shapes and texture but an interesting relationship has been observed between the Golden Ratio and the leaf structure. In the present study, two parameters, length and width of leaves of several common trees have been measured and the averages are recorded. Surprisingly, it is observed that in the majority of cases (more than 70%), the ratio of length and the width of the different tree leaves lies in between 1.5 to 1.8, which is nearly equal to Golden Ratio 1.618 .... A part of the results is presented in Table 2.

Table 2 (below): Represents the local names, scientific names, the length and width of the leaves of some plants.

Serial No.	Local name	Scientific Names	Average Length of leaf (L) cm	Average width of the leaf (B) cm	Ration of L and B (L/B)	Observation
1.	Siuli	*Nyctanthes arbor-tristis*	10.3	6.3	1.6349	G. Ratio
2.	Kori flower	*Tabernaemontana*	7	4.29	1.6317	G. Ratio
3.	Jaba	*Hibiscus rosa-sinensis*	9.9	6.1	1.6229	G. Ratio
4.	Gulancha (Giloi)	*Tinospora cordifolia*	7.5	4.7	1.5957	G. Ratio
5.	Tulsi	*Ocimum sanctum Linn*	4.3	2.6	1.6538	G. Ratio
6.	Kachupata	*Colocasia esculenta*	21.5	13.5	1.5925	G. Ratio
7.	Wild pant	*Unknown*	6.4	3.7	1.7297	G.Ratio
8.	Akanda	*Calotropis giganta*	14.6	7.9	1.8481	G. Ratio
9.	Land Lotus/ Cotton Rose/ Sthal- Padma	*Hibiscus acetosella*	18.5	12.1	1.5289	G. Ratio
10.	Sim	*Phaseolus vulgaris*	11.5	7.6	1.5131	G. Ratio
11.	Boroi	*Ziziphus mauritiana*	5.3	3.2	1.6562	G. Ratio
12.	Aparajita	*Clitoria terntea*	4.4	2.9	1.5172	G. Ratio

For better understanding, the photographs of the leaves of the above-mentioned plants are presented below.

3. Conclusion

This study presents only two case studies regarding the relation between the two kingdoms, mathematics and the plants. The beauty of mathematics is that it seems to be inherent in plants. The first empirical study shows that how the Fibonacci sequence is followed by the flower petals. A fascinating observation is that maximum flowers have 5 petals. Most probably, it is due to the fact that the pentagon structure is one of the most stable structures. The second study presents how the structure of most of the leaves follow the Golden Ratio. Obviously, some exceptions are present everywhere to establish the regularity.

Refernces

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