Problem Section 3

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We are starting a new section, which will have problems without solutions. We ask the solutions from the readers which we will publish in the subsequent issues. All solutions should preferably be typed in LaTeX and emailed to the editor. If you would like to propose problems for this section then please send your problems (with solutions) to the above mentioned email address, preferably typed in LaTeX. Each problem or solution should be typed on separate sheets. Solutions to problems in this issue must be received by *20 September, 2021.* If a problem is not original, the proposer should inform the editor of the history of the problem. A problem should not be submitted elsewhere while it is under consideration for publication in Ganit Bikash. Solvers are asked to include references for any non-trivial results they use in their solutions.

Problem 7. *Proposed by Ayan Nath (Tezpur) and Abhishek Jha (Delhi).*

Find all monotonic functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(-a^3 + b^3 + c^3) + f(a^3 - b^3 + c^3) + f(a^3 + b^3 - c^3) + 24f(abc) = f(a^3 + b^3 + c^3),$$

for all $a, b, c \in \mathbb{R}$. Bonus: solve it without the monotonicity constraint.

Problem 8. *Proposed by Anupam Saikia (Indian Institute of Technology Guwahati).*

Let G be a group containing no subgroup of index 2. Show that any subgroup of index 3 must be normal. (*The question requires knowledge of group homomorphism, group action and permutation group S_3.*)

Problem 9. *Proposed by B. Sury (Indian Statistical Institute, Bangalore).*

Consider the Fibonacci numbers defined by $F_1 = 1 = F_2$ and $F_{n+1} = F_n + F_{n-1}$. Prove that

$$F_n = \prod_{r=1}^{[(n-1)/2]} (3 + 2\cos(2\pi r/n)).$$

Use this to deduce that F_k divides F_l if and only if k divides l.

Editor's Note: No solutions have been received from the floor for Problems 1, 2 and 3 of Volume 67. We keep them open until 20 June, 2021 and request solutions from the readers.