Problem Section 2

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We have started this section from the last issue, which will have problems without solutions. We ask the solutions from the readers which we will publish in the subsequent issues. All solutions should preferably be typed in LaTeX and emailed to the editor. If you would like to propose problems for this section then please send your problems (with solutions) to the above mentioned email address, preferably typed in LaTeX. Each problem or solution should be typed on separate sheets. Solutions to problems in this issue must be received by *20 June, 2021*. If a problem is not original, the proposer should inform the editor of the history of the problem. A problem should not be submitted elsewhere while it is under consideration for publication in Ganit Bikash. Solvers are asked to include references for any non-trivial results they use in their solutions.

Problem 4. *Proposed by Anupam Saikia (Indian Institute of Technology Guwahati).*

Let F_n be the Fibonacci sequence defined as

$$F_1 = 1,\ F_2 = 1,\ F_n = F_{n-1} + F_{n-2} \quad n \geq 2.$$

Show that any prime p divides $F_{2p(p^2-1)}$.

Problem 5. *Proposed by Mangesh Rege (Shillong).*

Do there exist infinitely many natural numbers n, such that $n^2 + 1$ divides $n!$?

Problem 6. *Proposed by B. Sury (Indian Statistical Institute, Bangalore).*

If $p(x)$ and $q(x)$ are polynomials, with $\deg p(x) \leq \deg q(x) - 2$ and if the roots $\alpha_1, \alpha_2, \cdots, \alpha_n$ of $q(x)$ are distinct, then prove that

$$\sum_{i=1}^{n} \frac{p(\alpha_i)}{q'(\alpha_i)} = 0.$$

Editor's Note: Problem 3 of Volume 67, proposed by B. Sury also appeared as a problem in the Indian National Mathematical Olympiad (INMO) 2019. The INMO problem was also proposed by B. Sury.