Freeman Dyson: A Short Portrait

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Freeman Dyson was different things to different people. For some he was a maverick physicist who scaled heights without ever bothering to complete a formal PhD degree. For others still, he was a brilliant mathematician whose insights and tenacity have enriched the field many times over. And for some others, he was a brilliant essayist, reviewer of good books and writer of flowing prose. It would be difficult and even foolish to attempt to capture the varied life and times of Dyson in a short space, more so because the author does not feel capable enough to do so. Instead, we provide a very short portrait of the life of Freeman Dyson without focusing on any particular area of his research.

Freeman was born on 15th December, 1923 which made him over 96 years old when he died in the beginning of this year on 28 February, 2020. He was a precocious child and went to Trinity College, Cambridge after completing his earlier years at Winchester College. At Trinity he had the good fortune to learn and work with both Paul Dirac, the English Nobel prize winning physicist known for his eponymous equation and G. H. Hardy, known to Indians more famously as the mentor of Srinivasa Ramanujan, but who at that time was the preeminent English mathematician working in areas of analysis and number theory.

It would probably be an understatement to say that Dyson had an unusual and interesting career graph. He worked for the Royal Air Force?s Bomber Command during World War II before graduating with a BA in Mathematics in 1945. Around this time, Dyson came up with the conjectured rank and crank of a partition to explain some of the striking congruences found by Ramanujan. We take this brief digression into mathematics to show just one piece of Dyson's work which has fuelled much research.

Srinivasa Ramanujan is known for his exemplary work related to integer partitions. We know that each positive integer can be written as sums of smaller positive integers. This notion is formalized in mathematics by calling such smaller sums as partitions of the bigger number. By a partition of a positive integer, say n mathematicians mean a sequence of non-increasing positive integers, say $\lambda_1, \lambda_2, \ldots, \lambda_k$ such that the sum of all these λ_i's equals to n. For instance, we can write the integer 4 in the following ways 4, $3+1$, $2+2$, $2+1+1$ and

1+1+1+1, which means that there are 5 partitions of the integer 4. Mathematically, we write this as $p(4) = 5$. Among the many properties of this partition function that Ramanujan studied, the most easily appreciable ones are his three famous congruences, which states for any positive nonnegative integer k we have

$$\begin{aligned} p(5k+4) &\equiv 0 \pmod{5}, \\ p(7k+5) &\equiv 0 \pmod{7}, \\ p(11k+6) &\equiv 0 \pmod{11}. \end{aligned}$$

Here by the symbolic notation $a \equiv b \pmod{c}$ we mean that $a - b$ is divisible by c, for integers a, b and c. This really means that $a - b = c \cdot \ell$ for some ℓ and this in turn says that $a - b$ can be divided into c classes of size ℓ. We can very easily verify the first congruence from our observation that $p(4) = p(0 \cdot 5 + 4) = 5 \equiv 0 \pmod{5}$.

Dyson wanted to explain the above congruences combinatorially. The nature of the congruences suggest that the partitions of $5k + 4$ is divided into five equinumerous classes, while that of $7k + 5$ is divided into seven classes and those of $11k + 6$ into eleven classes. Dyson wanted to isolate a statistic on the partitions which would give rise to these classes. For the first two cases he came up with the concept of *rank of a partition* and for the third he conjectured the existence of a statistic called the *crank of a partition* to explain the third Ramanujan congruence. The rank of a partition, also sometimes called the Dyson rank is defined as the difference between the largest part of the partition and the number of parts of the partition. For instance, in our example of the partitions of 4, the respective ranks are 3, 1, 0, -1 and -3 and we see that indeed there are five separate classes with distinct ranks in each class. This example is not particularly enlightening, we encourage the reader to check the same for the partitions of $p(9)$. Dyson observed that the ranks of partitions of n takes the value from one of $n-1, n-3, \ldots, 2, 1, 0, -1, -2, \ldots, 3-n, 1-n$. He defined the quantity $N(m, n)$ to be the number of partitions of n with rank m, and the quantity $N(m, q, n)$ to be the number of partitions of n whose rank ℓ satisfies $\ell \equiv m \pmod{q}$. What Dyson conjectured was that

$$\begin{aligned} N(0,5,5k+4) = N(1,5,5k+4) = N(2,5,5k+4) \\ = N(3,5,5k+4) = N(4,5,5k+4), \end{aligned}$$

and

$$\begin{aligned} N(0,7,7k+5) = N(1,7,7k+5) = N(2,7,7k+5) \\ = N(3,7,7k+5) = N(4,7,7k+5) = N(5,7,7k+5) \\ = N(6,7,7k+5). \end{aligned}$$

Dyson could not come up with such a natural explanation of the third congruence of Ramanujan, so he said that there must exist another partition statistic, which he called *crank of a partition* that would explain the third congruence. The above conjectures were proven by Atkin and Swinnerton-Dyer in 1954, while the elusive crank in its most natural form was found by Andrews and Garvan in 1988. We do not go into more details about this aspect, but we just mention that the study of ranks and cranks for different partition functions and their properties is an active area of research in number theory.

Coming back to the story of Dyson now, after this digression we find Dyson being awarded a Commonwealth scholarship which took him to Cornell University in the USA where he worked with another Nobel prize winning physicist, Hans Bethe for his PhD. But Dyson never formally got his PhD and later on came to be known as 'the Rebel without a PhD'. Dyson had by now shifted from mathematics to physics, more precisely to mathematical physics. Stories about his dexterity in using mathematics abound and are a source of awe-inspiring joy and admiration, so it was not a surprise that he choose mathematical physics as his vocation during this period.

It would be too numerous to recount all of the major achievements of Dyson in physics, ranging from his work in quantum mechanics, statistical physics, astrophysics to even nuclear physics and engineering. But we must mention his work in quantum electrodynamics (QED). It is not easy to explain what QED is without going into some amount of college level physics, so we will just use the term without going into the technicalities. One could however think of the subject as the quantum field theory version of classical electrodynamics, which studies the electromagnetic force. In the late 1940s, two different approaches of studying QED came up, one was by the physicist Richard Feynman who used diagrams, now known as Feynman diagrams to explain interactions between subatomic particles. The other approach was a much more mathematical approach found by physicist Julian Schwinger. Unbeknownst to them, Japanese physicist Shinichiro Tomonaga also came up with an equivalent formulation which didn't reach the USA in those immediate post-WWII years. The three of them shared the 1965 Nobel prize in Physics for this work. The three approaches of studying the same phenomenon worried some physicists. It took the brilliance of Dyson to show that in fact, the three approaches of Feynman, Schwinger and Tomonaga were mathematically equivalent. This piece of brilliant work by Dyson immediately catapulted him to the higher echelons of physicists where he stayed for the rest of his life.

Shortly after this, Dyson received an offer from Robert J. Oppenheimer to join the Institute for Advanced Study, one of the world's preeminent centres of research in theoretical sciences. Dyson gladly accepted this invitation and he stayed in the institute for the rest of his life. The institute is unique in the sense that there are no teaching duties for the faculty and there is academic freedom to pursue each individual's own program of research. Dyson branched out into several areas during his tenure at the institute, including a somewhat ambitious project called "Project Orion" to design an atomic spaceship capable of riding a wave of controlled nuclear pulses into deep space. It was during this years that Dyson started to write more frequently about his world view of science, about books he enjoyed and about his own life. This has resulted in several important books chronicling Dyson as a scientist, as a reader and as an essayist.

It would come as no surprise that a distinguished scientist such as Dyson has been awarded with many prestigious prizes and awards in his lifetime. These includes the Fellowship of the Royal Society at the age of 30 and the membership of the National Academy of Sciences, the Henri Poincaré Prize of the International Mathematical Physics Congress (2012), the Templeton Prize for Progress in Religion (2000), the Enrico Fermi Award of the U.S. Department of Energy (1995), the Wolf Prize in Physics (1981) and many others. On 28th of February, 2020 Dyson breathed his last and with him came down one of the last titans of the first and middle half of the last century. Dyson lived a varied life, in terms of his research as well as his controversial opinions of some matters such as climate change. A colourful personality and a versatile person, his writings are a source of boundless joy and his enthusiasm for science is almost infectious. The list of things named after Dyson is indeed a testament to his rich oeuvre and would no doubt provide many more generations of mathematicians and physicists food for thought.