

Assam Academy of Mathematics

MATHEMATICS OLYMPIAD

26th May, 2019

Category-I : (For Classes- V & VI)

Total Marks : 100 Time : 10.00 AM – 1.00 PM

1. abcde is a five digit number. Two six digit numbers are formed by putting 9 to the left and right of it respectively. If the former is equal to 4 times of the latter, find $a+b+c+d+e$. 7

Soln.

By given condition,

$$9abcde = 4 \times abcde9$$

$$\begin{array}{r} \text{i.e.} \quad a \ b \ c \ d \ e \ 9 \\ \qquad \qquad \qquad \times 4 \\ \hline \end{array}$$

$$9 \ a \ b \ c \ d \ e$$

Comparing with digits in the bottom row–

$$e = 6, d = 7, c = 0, b = 3, a = 2$$

Thus abcde is 23076

$$\begin{aligned} \text{Therefore } a+b+c+d+e &= 2+3+0+7+6 \\ &= 18 \end{aligned}$$

2. If you write all the first 100 natural numbers side by side in its natural order a large number will be formed. Now find the number of digits in the number so formed. Also find the sum of all the digits in the number. 7

Soln.

$$\text{No. of digits from 1 through 9} = 9$$

$$\text{No. of digits from 10 through 99} = (99-9) \times 2 = 180$$

(2)

$$\text{No. of digits in 100} = 3$$

$$\begin{aligned} \text{Total no. of digits in} \\ \text{numbers from 1 through 100} &= 9+180+3 \\ &= 192 \end{aligned}$$

Next,

The no. of 0's is 11

The no. of 1's is 21

The no. of 2's is 20

The no. of 3's is 20

The no. of 4's is 20

The no. of 5's is 20

The no. of 6's is 20

The no. of 7's is 20

The no. of 8's is 20

The no. of 9's is 20

Hence sum of all digits of the number composed of serial arrangement of numbers from 1 through 100

$$\text{is } 11 \times 0 + 21 \times 1 + 20 \times (2+3+4+5+6+7+8+9)$$

$$= 21 + 20 \times 44$$

$$= 21 + 880$$

$$= 901$$

3. If the sum of seven consecutive natural numbers is 126, find the numbers. 6

Soln.

$$126 \div 7 = 18$$

So, the seven numbers might be around 19 let us try with 15, 16, 17, 18, 19, 20, 21

$$\text{The sum of these numbers} = 126$$

(3)

Thus, the consecutive numbers are

15, 16, 17, 18, 19, 20 and 21

4. Assign appropriate digits to the letters involved in the following two additions so that both of them remain correct in their digital values also. 7

$$\begin{array}{r} \text{ONE} \\ + \text{ONE} \\ \hline \text{TWO} \end{array} \quad \begin{array}{r} \text{ONE} \\ + \text{FOUR} \\ \hline \text{FIVE} \end{array}$$

$$E+R=E \quad \Rightarrow R=0$$

$$E+E=0 \quad \Rightarrow 0 \text{ is even, but } 0+0 \text{ is less than } 10$$

Hence 0 is 4 or 2

$$\text{If } 0=2, \quad E=1 \text{ or } E=6$$

But it can be seen that $E=1$ is not possible

Therefore $E=6$

After a series of trial and error we see that $N=3$ or $N=8$

$$\therefore \text{One} = 236 \text{ or } \text{One} = 286$$

When $\text{One} = 236$, we obtain the sums as

$$\begin{array}{r} 236 \\ 236 \\ \hline 472 \end{array}, \quad \begin{array}{r} 236 \\ \text{F}2\text{U}0 \\ \hline 9516 \end{array} \Rightarrow U = 8, F = 9$$

$$\text{Thus } \begin{array}{r} 236 \\ 236 \\ \hline 472 \end{array}, \quad \begin{array}{r} 236 \\ 9280 \\ \hline 9516 \end{array} \text{ give one solution.}$$

Again $\text{ONE} = 286$ gives in

$$\begin{array}{r} 286 \\ 286 \\ \hline 572 \end{array}, \quad \begin{array}{r} 286 \\ 92\text{U}0 \\ \hline 3496 \end{array} \text{ is } U = 1, F = 3$$

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(4)

The other solution is

$$\begin{array}{r} 286 \\ 286 \\ \hline 572 \end{array}, \quad \begin{array}{r} 286 \\ 3210 \\ \hline 3496 \end{array}$$

5. In the following addition sum, each of the ten digits is different and also the digits to be added in each column from top to bottom are in increasing order. Determine the digits in the sum 7

$$\begin{array}{r} \text{A} \\ + \text{B}4 \\ + \text{CDE} \\ \hline \text{FGHI} \end{array}$$

Soln.

A closer observation leads us to conclude that $C=9$ and 1 will be carried over to $C=9$ from $B+D$ to get $FG=10$

Thus the sum becomes

$$\begin{array}{r} \text{A} \\ \text{B}4 \\ 9\text{DE} \\ \hline 10\text{HI} \end{array}$$

Again $A < 4$ and $E > 4$, $A, 4, E$ being in increasing order

After few trial and error steps we arrive at

$$A = 3$$

$$E = 5$$

$$B = 7,$$

$$D = 8, H = 6, I = 2$$

i.e. sum is

$$\begin{array}{r} 3 \\ 74 \\ 985 \\ \hline 1062 \end{array}$$

6. In a secret message if 'CDUQNWVG' is to be decoded as 'ABSOLUTE' how would you code the message 'NO WAY' 6

Soln.

According to the code

$$A \rightarrow C$$

$$B \rightarrow D$$

$$S \rightarrow U$$

$$L \rightarrow N$$

$$U \rightarrow W$$

$$T \rightarrow V$$

$$E \rightarrow G$$

Therefore

$$N \rightarrow P$$

$$O \rightarrow Q$$

$$W \rightarrow Y$$

$$A \rightarrow C$$

$$Y \rightarrow A$$

Hence, NO WAY is to be coded as PQ YCA

7. I am a three digit square number. If you divide me and the sum of my three digits by 3 and 5 you will find the remainder 1 in each case. Who am I? 6

Soln.

Three digit square numbers are–

100, 121, 144, 169, 196, 225, 256, 289, 324, 361

400, 441, 484, 529, 576, 625, 676, 729, 784, 841,

900, 961

By condition, my units digit must be 1 more than 0 or 5 since I am divisible by 5. Therefore my units place will be 1 or 6. Hence, I am one of–

121, 196, 256, 361, 441, 576, 676, 841 and 961

Now

$1+9+6 = 16$ satisfies the other condition also.

$9+6+1 = 16$ also satisfies the other condition

Hence, I am 196 or 961

8. Follow the pattern given below and supply at least five terms to continue the pattern further. 6

4, 6, 8, 9, 10, 12, 14, 15, 16, , , , .

Soln.

The terms in the sequence are all composite numbers. In other words the prime numbers are not to be considered.

So, next five terms after 16 are–

18, 20, 21, 22, 24.

9. Supply the missing figures in the following multiplication 7

$$\begin{array}{r} * 5 2 * \\ \quad 3 * \\ \hline 7 * * 2 \\ * * * 8 \\ \hline 8 * * * * \end{array}$$

Soln.

For convenience *'s are replaced by letters like

$$\begin{array}{r} a 5 2 b \\ \quad 3 c \\ \hline 7 d e 2 \\ \quad f g h 8 \\ \hline 8 j k l m \end{array}$$

From the multiplication, we can see

$3 \times *$ leaves 2 in the unit place

Hence * must be 4

Sum becomes–

$$\begin{array}{r} a 5 2 4 \\ \quad 3 c \\ \hline 7 5 7 2 \\ \quad f g h 8 \\ \hline 8 j k l m \end{array}$$

(7)

$$\text{Now, } c \times 4 = 8 \quad \therefore c = 2$$

Product becomes

$$\begin{array}{r} 2524 \\ \underline{32} \\ 7572 \\ \underline{5048} \\ 80768 \end{array}$$

10. Find the greatest number of four digits and the least number of five digits which when divided by 789 leave a remainder 5 in each case.

Soln.

The greatest four digit number is 9999.

Now 9999 divided by 789 leaves quotient 12 and 531 as remainder. Thus, the greatest four digit number divisible by 789 is $9999 - 531 = 9468$. Required greatest four digit number that leaves remainder 5 upon divided by 789 is $9468 + 5 = 9473$

$$\begin{array}{r} 12 \\ 789 \overline{) 10000} \\ \underline{789} \\ 2110 \\ \underline{1578} \\ 532 \end{array}$$

Again the least five digit number 10000 divided by 789 leaves remainder 532. But $789 - 532 = 257$

Therefore, the least five digit number divisible by 789 is $10000 + 257 = 10257$.

Hence, the least five digit number is leaving remainder 5 when divided by 789 is $10257 + 5 = 10262$

Required numbers are 9473 and 10262

(8)

11. A sum of Rs. 22000 was distributed among 60 students such that each senior student gets Rs. 500/- while each junior student gets Rs. 300/-. Find the numbers of senior and junior students among them. 6

Soln.

The amount required to distribute among 60 students at the rate of Rs. 300 per student is $\text{Rs. } 300 \times 60 = \text{Rs. } 18000$
So, $\text{Rs. } 22000 - \text{Rs. } 18000 = \text{Rs. } 4000$ can be distributed to senior students at the rate of Rs. 200 per student.

Thus the number of senior student is $4000 \div 200 = 20$
Hence the number of junior students is 40

Senior students 20, Junior students 40

12. A salesman bought a certain number of eggs for Rs. 186/- and sold some of them for Rs. 66/- without any profit. Show that he was still left with at least 20 eggs. 7

Soln.

We have $186 - 66 = 120$

$$\text{and } \begin{array}{r} 2 \overline{) 66, 120} \\ 3 \overline{) 33, 60} \\ 11, 20 \end{array}$$

The salesman can sell at a maximum rate Rs. 6 per egg.
Hence the minimum number of eggs left with the salesman is $120 \div 6 = 20$

i.e. The salesman has at least 20 eggs for selling.

13. A vessel contains a mixture of 30 litres of water and milk in the ratio 7:3. How much milk must be added to the mixture so that the ratio of water and milk becomes 3:7? 7

Soln.

Water in the vessel is $\frac{7}{10} \times 30 = 21$ liters

(9)

Milk in the vessel is $\frac{3}{10} \times 30 = 9$ liters

After adding milk to the mixture,

Water : Milk = 3:7

i.e. 21 : Milk = 3:7

or Milk : 21 = 7:3

\therefore Milk = $\frac{7}{3} \times 21 = 49$ liters

Amount of milk to be added is $49 - 9 = 40$ litres.

14. Find the least square number which is divisible by 10, 16 and 24. 7

Soln.

$$\begin{array}{r}
2 \overline{)10, 16, 24} \\
2 \overline{)5, 8, 12} \\
2 \overline{)5, 4, 6} \\
5, 2, 3
\end{array}$$

\therefore L C M of 10, 16 and 24

is $2 \times 2 \times 2 \times 5 \times 2 \times 3$

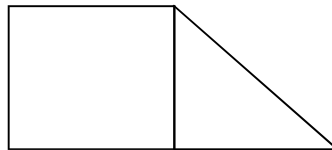
= $2^2 \times 2^2 \times 5 \times 3$

Hence the least square number divisible by 10, 16 and 24

is = $2^2 \times 2^2 \times 5^2 \times 3^2 = 16 \times 425$

= 6800

15. The shape shown below is that of a square attached to half of another square of equal size divided diagonally. Can you divide it into four pieces all of precisely the same size and shape? 7



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(10)

Soln.

Four pieces of same shape and size can be done as follows—

