Assam Academy of Mathematics

MATHEMATICS OLYMPIAD

September 10, 2017 (Sunday)

Category-II : For classes- VII & VIII

Total Marks : 100 Time : 11.00 AM – 2.00 PM

1. Two numbers when divided by a certain divisor have remainders 3 and 4 respectively. When the two numbers are added and their sum is divided by the same divisor the remainder is 2. What is the divisor ? 5

Ans : The two numbers, when divided by a divisor leave remainders 3 and 4.

When we add the two numbers, their remainders add up to 3+4=7

But when divided by the original divisor this sum leaves remainder equal to 2.

Since 7=5×1+2, the divisor must be 5.

2. Find the number nearest to the number 9085345 which must be added to 465 to make the sum exactly divisible by 989. 5

Ans : 9085345=989×9186+391

Now 465–391=74

Adding 74 to the given number we have 9085345+74=9085419

which is the nearest of 9085345 divisible by 989.

But, the number 9085345 is to be obtained by adding 465. Hence the required number is 9085345–465=9084880.

3. Find the digit values of x and y in order that the number 5x16y5 may be divisible by 275. 5

Ans : 275=25×11

Thus 5x16y5 is divisible by 275 implies that it is divisible by both 25 and 11.

But the number will be divisible by 11 it 5–x+1–6+y–5 = y–x–5 is either zero or divisible by 11.

Also 5x16y5 is divisible by 25 if 6y5 is divisible by 25. Then y is either 2 or 7.

When y=2, y–x–5 = 2–x–5 = 0

i.e. x = 3

When y=7, y–x–5 = 7–x–5 = 0

i.e. 2–x = 0

i.e. x = 2

Taking x=3, y=2, we have

531625 which is not divisible by 11 since 5–3+1–6+2–5=–6 which is neither zero nor divisible by 11.

Taking x = 2, y = 7, we have

521625 which is divisible by 275.

Thus x = 2, y = 7.

4. Find the missing terms in the following arrangement–

1, 1, 2, 5, 29, * 866, *, ... 5

Ans : Here every term after 2nd is the sum of squares of the preceding two terms.

Thus 3rd term = (1st term)^2 + (2nd term)^2

$= 1^2+1^2$

$= 2$

4th term $= (2nd\ term)^2+(3rd\ term)^2 = 1^2+2^2=5$

5th term $= 2^2+5^2 = 29$

6th term $= 5^2+29^2=866$

7th term $= 29^2+866^2=750797$

Note– The sequence should have run as

1, 1, 2, 5, 29, 866, *, ...

So missing term is 750797.

5. Find the last digit (unit's digit) of the sum.

$1^2+2^2+3^2+4^2+$ .....+ 99^2 5

Ans : $1^2+2^2+3^2+4^2+$ .....+ 99^2

Unit's digit of $1^2,2^2,3^2,4^2,5^2,6^2,7^2,8^2,9^2$ are 1,4,9,6,5,6,9,4,1 respectively

When added these digits will give the sum 1+4+9+6+5+6+9+4+1 = 45

Now the sum 45 will be repeated ten times as we take the whole sum.

$(1^2+2^2+.....+9^2)+10^2+(11^2+.....19^2)+20^2+(.....)$

$+(91^2+92^2+.....99^2)$

Hence unit's digit in the sum will be the unit's digit of 45×10=450.

i.e. unit's digit of the sum is 0.

6. After dividing a certain number by 537 a boy obtained 12578 as quotient. If the digit 8 is wrong and if the true remainder and the digit in the unit's place of the dividend be respectively 459 and 8 find the dividend and the correct quotient. 5

Ans : By the given inputs, the required dividend will be

$537\times12578+459$

$= 6754386+459$

$= 6754845$

But 8 in the quotient 12578 is wrong and unit's digit in the dividend

should be 8. Then the unit's digit in 6754386 must be 9 and then 8 in the 12578 must be 7.

Thus actual dividend must be

$537\times12577+459$

$= 6753849+459$

$= 6754308$

7. Supply the missing figures in the following simplification– 5

$$43\frac{89}{\odot\odot} \div \odot \frac{5}{11} = 5\frac{8}{9}$$

Ans : Let us write the relation as

$$43\frac{89}{ab} \div c\frac{5}{11} = 5\frac{8}{9}$$

where ab = 10a+b

Simplifyingm, we have

$$\frac{43(10a+b)+89}{10a+b} \times \frac{11}{11\times c+5} = \frac{53}{9}$$

Clearly, 10a+b and 11×c+5 are both two digit numbers and in the right land side we have denominator q which is a single digit number. There fore 10a+b must be divisible by 11 an 43 (10a+b)+89 must be divisible by 11×c+5. The relatiouship helps

us to guess.

That $10a+b = 11\times9 = 99 = 10\times9+9$

Thus $a = b = 9$

Then $43\times(10a+b)+89$

$= 43\times99+89$

$= 4257+89$

$= 4346.$

Thus $4346 = 53\times(11\times c+5)$

i.e. $11\times c+5 = \frac{4346}{53} = 82$

$\Rightarrow \quad 11 \times c = 77$

$\Rightarrow \quad c = 7$

Thus the missing numbers are 99 and 7.

8. The first January, 1929 was on Tuesday; What day of the week was October 1st of the same year ? 5

Ans : No. of days in January, 1929 = 31

No. of days in Febuary, 1929 = 28

No. of days in March, 1929 = 31

No. of days in April, 1929 = 30

No. of days in May, 1929 = 31

No. of days in June, 1929 = 30

No. of days in July, 1929 = 31

No. of days in Augest, 1929 = 31

No. of days in September, 1929 = 30

No. of days in October, 1929 = 1

Total No. of Days = 274

No. of days from January 1 to October 1 of 1929 is

$274 = 39\times7+1$

Since January 1 of 1929 was a Tuesday, October 1 of the same year will be a Wednesday.

9. Find the digit values of the letters A, B, C, D and E from the following sum and determine the L.C. M of CD and AE. 5

D	CD, AE
C	D, C
	B, A

Ans : From the sum, it is clear that D×D = CD

Hence D is 5 or 6

But D if is 5 then D×D=25 = CD

i.e. C = 2

But C×B = D

i.e C=2 Divides D=5

which is impossible.

Hence D=6, so that D×D = 36 = CD

i.e. C = 3 and C = 3 Divides D = 6.

But C×B = D i.e. 3×B = 6

i.e. B = 2

Again D×C = AE

i.e. 6×3 = 18

i.e. A = 1, E = 8 So That C×A = C

Thus CD = 36 and AE = 18 with B = 2

Now LCM (CD,AE) = LCM (36, 18) = D×C×B×A

= 6×3×2×1

= 36

10. Divide ` 52 between A, B and C in such a manner that B may receive $.\dot{3}$ of A and C, $.\dot{3}$ of B. 5

Ans : C receives $.\dot{3} = \frac{1}{3}$ of B

B receives $.\dot{3} = \frac{1}{3}$ of A

Therefor C receives $\frac{1}{3}$ of $\frac{1}{3}$ of A

$= \frac{1}{9}$ of A

The amount of A, B and C together is Rs. 52/-

i.e. $A + \frac{1}{3}A + \frac{1}{9}A = 52$

i.e. $\left(1 + \frac{1}{3} + \frac{1}{9}\right)$ of $A = 52$

i.e. $\frac{13}{9}$ of $A = 52$

Hence part of $A = 52 \times \frac{9}{13} = 36$, Then B's part is $\frac{1}{3} \times 36 = 12$ and

C's part is $\frac{1}{9} \times 36 = 4$.

Therefore the amounts of A, B And C are respectively 36, 12 and

11. Find the ratio of the least and the greatest four digit numbers which are exactly divisible by 34. 5

Ans : The least four digit number is 1000.

1000 divided by 34 gives quolient as 29 and remainder 14.

If we add $34 - 14 = 20$ to 1000, then we obtain the least four digit number divisible by 34. That is the least four digit number divisible by 34 is 1020.

Again the greatest four digit number is 9999 and 9999 divided by 34 gives quotient equal to 294 and remainder equal to 3.

Hence 9999–3 = 9996 is the greatest four digit number divisible by 34.

Hence the required ratio is

$$\frac{1020}{9996} = \frac{34 \times 30}{34 \times 294} = \frac{5}{49}$$

12. In the number 13579 insert a zero somewhere between 1 and 9 so as

(i) to make the greatest possible difference in the number.

Ans : The number 13579.

The insertion of 0 in between 7 and 9 will make the number greatest of all numbers obtained by inserting 0 at any other place.

Thus 135709 will be greatest of all numbers obtained by inserting 0 in any other place in between 1 and 9 of the given number. In this case differance in the number becomes greatest and it is equal to $135709 - 13579 = 122130$.

(ii) to make the least possible difference; and find the difference of these two differences. 5

Ans : Again, insertion of 0 in between 1 and 3 will make the number least of all numbers obtained by insertly 0 at any other position in between 1 and 9. Thus 103579 is the least of all numbers obtained by inserting 0 in between 1 and 9 of 13579. In this case $103579 - 13579 = 90000$ and this is the least difference occurned to the given number.

Hence required difference $= 122130 - 9000 = 32130$.

13. 90% of the boys of a school pass in English and 85% in Arithmetic, 150 boys pass in both the subjects and no boy fails in both. How many boys are there in the school? 5

Ans : Clearly 10% fails in English and 15% fails in Arithmetic.

So, if there were 100 students in the school, then 10 students failed in English and 15 students failed in Arithmetic. Since no student failed in both subjects, total number of failed student is 25. In other words, total number of students passed is 75 out of 100.

i.e. 75% of total students = 150

Hence total number of student must be $150 \times \frac{100}{75}$

$= 200$

14. The price of sugar being raised 50%, by how much percent must a man reduce his consumption of that article so as not to increase his expenditure ? 5

Ans : Let monthly consumption of sugar be x and the original price be y.

With hike of 50%, new price will be $y + \frac{50y}{100} = y + \frac{y}{2} = \frac{3y}{2}$

At a price of $\frac{3y}{2}$, the amount of sugar is x.

Hence at a price of y amount of sugar is $\frac{x}{\frac{3y}{2}} \times y$

$= \frac{2x}{3}$

Hence the man must reduce the amount of sugar by

$x - \frac{2x}{3} = \frac{x}{3}$

Hence percentage of redution is $\frac{\frac{x}{3}}{x}\times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$.

15. A clock indicates correct time when the hands are together between 2 and 3 0' clock; if it had been losing 2 minutes every hour what time did it indicate at 12 noon? 5

Ans : Let the hands of the clock be togethar at x minutes e past 2.

When the hour hand advances 5 readings, the minute hand advances 60 readings. in the dial. i.e. the minute hand advances 55 readings ahead of hour hand in 60 minutes.

Hence the minute hand advances 1 reading ahead of hour hand in $\frac{60}{55}$ minutes.

Now at 2 o' clock, hour hand is 10 readings ahead of minutes hand. Hence before the minute hand meet the hour land, it has to cross 10 readings and the requied time will be $10\times\frac{12}{11}$ minutes

$$= \frac{120}{11} = 10\frac{10}{11} \text{ minutes}$$

In other wonds, the clock shows correct time at 2 0' clock past $10\frac{10}{11}$ minutes.

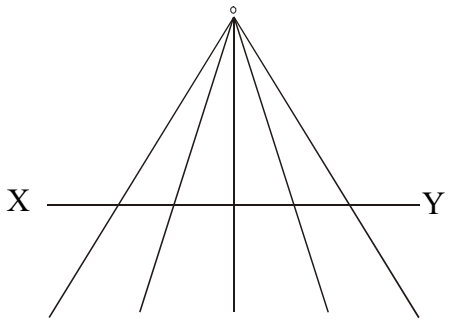
But the clock was losing 2 minutes per hour.

i.e. in 60 minutes it was losing 2 minutes.

Therefore in $\frac{120}{11}$ minutes, it was lasing $\frac{2}{60}\times\frac{120}{11} = \frac{4}{11}$

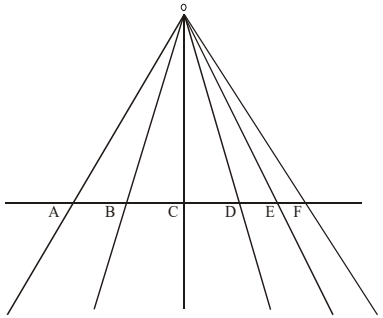
Hence at 12 noon, it was showing 12 hour + $\frac{4}{11}$ minutes.

16. Six lines drawn from a given point 0 intersect another line XY as given in the following figure. Find the total number of triangles formed. 5



Ans : There is a flaw in the question in that the statement of the

problem does not tally with the adjoint figure. One has to cousider six lines through 0 intersecting XY in stead of five as given in the figure.



The number of triangles will be equal to the number of ways two paints can be selected from six points viz A, B, C, D, E and F.

But two points in this case can be selected as

$\{A,B\},\ \{A,C\},\ \{A,D\},\ \{A,E\},\ \{A,F\},\ \{B,C\}$

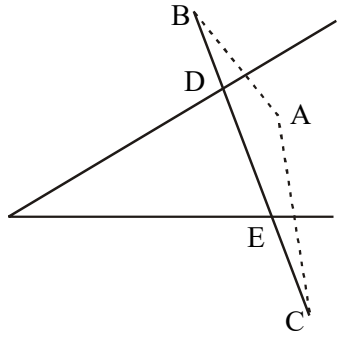
$\{B,D\},\ \{B,E\},\ \{B,F\},\ \{C,D\},\ \{C,E\},\ \{C,F\}$

$\{D,E\},\ \{D,F\},\ \{E,F\}$

Thus the number of triagles formed is 15.

17. The point A, inside an acute angle, is reflected in either side of the angle to obtain points B and C. Line segment BC intersects the sides of the angle at D and E. Show that–

$\frac{BC}{2} > DE$



5

(Note that the length of any side of a triangle is less than semi perimeter of the triangle).

Ans : Since B is a reflection of A on the arm through D, therefore

$AD = BD.$

Similary $AE = EC$

Now, $BC = BD + DE + EC$

$< BD + AD + AE + EC.$ (Since $DE < AD + AE$)

$= 2\,(BD + EC)$

$\therefore \quad \frac{BC}{2} < BD + EC = BC - DE.$

Thus $DE < BC - \frac{BC}{2} = \frac{BC}{2}.$

18. Solve the following equation for integer values of x and y.

$(2x+y)\ (5x+3y) = 7$ 5

Ans : We have $(2x+y)(5x+3y) = 7$

Hence x, y being integers, $2x+y$ and $5x+3y$ are both integers and their product is 7 whose only factors are 1, 7 or –1, –7.

Then we have the following cases–

$2x+y = 1, \quad 5x+3y = 7$ .....(i)

$2x+y = 7, \quad 5x+3y = 1$ .....(ii)

$2x+y = -1, \quad 5x+3y = -7$ .....(iii)

and $2x+y = -7, \quad 5x+3y = -1$ .....(iv)

From(i) $x = -4, \quad y = 9$

From(ii) $x = 20, \quad y = -33$

From(iii) $x = 4, \quad y = -9$

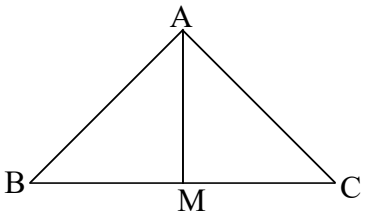
From(iv) $x = -20, \quad y = 33$

Hence solutions are

$(-4, 9), (20, -33), (4, -9), (-20, 33)$

19. In a triangle ABC the median AM is longer than half of BC. Prove that the angle BAC is acute. 5

Ans : The angle opposite to a longer side is greater than the angle opposite to a shorter side.



Given that $AM > \frac{1}{2}BC = BM = CM$

Now $AM > BM$

$\Rightarrow \quad \angle ACB > \angle BAM = \delta\frac{1}{2}\angle A$

Similarly $\angle ACB > \angle CAM = \frac{1}{2}\angle A$

$\therefore \quad ABC + \angle ACB > \frac{1}{2}\angle A + \frac{1}{2}\angle A = \angle A$

i.e. $180 - \angle A > \angle A$

i.e. $180 > 2\angle A$

$\therefore \quad \angle A > \frac{180}{2} = 90^0$

In other words $\angle A$ is acute.

20. Solve the Sudoku given below by inserting the numbers from 1 to 9 in the empty squares such that no number occupies more than once in any row or column. 5

7					4			8
8	4			1		3		
2				3			6	
							3	2
1		4		5	2			7
5		3						1
	5			2			1	
6							4	
	7	8			3		2	

Ans : The solution to the sudoku is as follows–

7	③	①	⑥	⑨	4	②	⑤	8
8	4	⑥	②	1	⑤	3	⑦	⑨
2	⑨	⑤	⑦	3	⑧	①	6	④
⑨	⑥	⑦	⑧	④	①	⑤	3	2
1	⑧	4	③	5	2	⑥	⑨	7
5	②	3	⑨	⑦	⑥	④	⑧	1
③	5	⑨	④	2	⑦	⑧	1	⑥
6	①	②	⑤	⑧	⑨	⑦	4	③
④	7	8	①	⑥	3	⑨	2	⑤