Assam Academy of Mathematics

MATHEMATICS OLYMPIAD

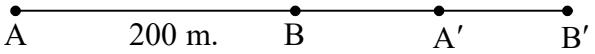
September 10, 2017 (Sunday)

Category-I : For classes- V & VI

Total Marks : 100 Time : 11.00 AM – 2.00 PM

1. (a) A thief is noticed by a policeman from a distance of 200 meters. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minutes? 3

Ans :



Let A, B be the original positions of the policeman and the thief respectively so that AB = 200 meters.

Let A', B' be the respective positions of the policeman and the thief after 6 minutes.

$$AA' = \frac{11000}{60} \times 6 = 1100 \text{ meters.}$$

$$BB' = \frac{1000}{60} \times 6 = 1100 \text{ meters.}$$

Dictance between the thief and the policeman is

$$A'B' = AB' - AA'$$

$$= AB + BB' - AA'$$

$$= 200 + 1000 - 1100 = 100 \text{ meters.}$$

(b) The salaries of A, B, C are in the ratio 2 : 3 : 5. If the increments of 15 %, 10 % and 20 % are allowed respectively in their salaries, then what will be the new ratio of their salaries? 3

Ans : Since the ratios of the salaries of A, B and C are given by

2 : 3 : 5

their actual salaries may be takan as 2K, 3K and 5K.

With 15%, 10% and 20% increase in their salaries, The new salaries of A, B, C are $2K+2K\times\frac{15}{100}$, $3K+3K\times\frac{10}{100}$, $5K+5K\times\frac{20}{100}$

i.e. $2K\left(\frac{105}{100}\right), 3K\left(\frac{110}{100}\right), 5K\left(\frac{120}{100}\right)$

The ratios of new salaries of A, B, C will be

$$2K\left(\frac{115}{100}\right) : 3K\left(\frac{110}{100}\right) : 5K\left(\frac{120}{100}\right)$$

$$= 2\times115 : 3\times110 : 5\times120$$

$$= 2\times23 : 3\times22 : 5\times24$$

$$= 23 : 33 : 60$$

(c) A number when divided by 296 leaves 75 as remainder. Find the remainder when the same number is divided by 37. 4

Ans : We have

$296 = 37 \times 8$

Also $75 = 37 \times 2 + 1$

296 being exactly divisible by 37, there is no change in the original remainder i.e. 75. But 75 has a remainder 1 when divided by 37. Hence, the required remainder, when the number is divided by 37 is 1.

2. (a) Find the missing term in the following sequence.
7, 26, 63, 124, 215, 342, (*), ... 3

Ans : Her

$7 = 2^3 - 1$

$26 = 3^3 - 1$

$63 = 4^3 - 1$

$124 = 5^3 - 1$

$215 = 6^3 - 1$

$342 = 7^3 - 1$

$\therefore \quad * = 8^3 - 1 = 511$

(b) If Z = 52 and ACT = 48 then BAT = ? 4

Ans : $Z = 52 = 2 \times 26$

$ACT = 48 = 2 \times (1+3+20)$

$\therefore \quad BAT = 2 \times (2+1+20) = 46$

(c) In a certain code language COMPUTER is written as RFUVQNPC. How will MEDICINE be written in that code language? 3

Ans : Observe the movements of the letters from right to left in "COMPUTER".

$R \rightarrow R$ (invariant)

$E \rightarrow F$ (one step forword)

$T \rightarrow U$ (one step forword)

$U \rightarrow V$ (one step forword)

$P \rightarrow Q$ (one step forword)

$M \rightarrow N$ (one step forword)

$O \rightarrow P$ (one step forword)

$C \rightarrow C$ (invariant)

For MEDICINE, the arrangement must go as follows–

$E \rightarrow E$ (invariant)

$N \rightarrow O$ (one step forword)

$I \rightarrow J$ (one step forword)

$C \rightarrow D$ (one step forword)

$I \rightarrow J$ (one step forword)

$D \rightarrow E$ (one step forword)

$E \rightarrow F$ (one step forword)

$M \rightarrow M$ (invariant)

Hence MEDICINE becomes – EOJDJEFM.

3. (a) A group of students decided to collect as many paise from each member of the group as is the number of members. If the total collection amounts to Rs 59.29, find the number of students. 3

Ans : Since the amount to be collected from each student is the same as the number of students, clearly, the number of students must be square root of 5929.

i.e. No. of students = $\sqrt{5929}$

= 77.

(b) In a class of 60 students where girls are twice that of boys, Laxmi ranked 27th from the top. If there are 9 boys ahead of Laxmi, how many girls are there after her rank? 4

Ans : Since girls are twice as many as the number of boys, and there are 60 of them altogether, there fore,

the number of boys = 20

the number of girls = 40

There are 9 boys ahead of Laxmi and Laxmi's position is 27. This means that there are 17 girls ahead of Laxmi.

Thus no. of girls after Laxmi = 40 – 18 = 22

(c) Two numbers a and b satisfy the equation

$56a = 65b$

Prove that a + b is composite. 3

Ans : Her $56a = 56b$

$56 = 7 \times 8$

$65 = 5 \times 13$

Thus, 56 and 65 have no common factor. Therefore the equality is possible only when.

$a = 65$ and $b = 56$

Thus a+b = 65+56 = 121 = 11×11 which is a composite number.

4. (a) Find the sum of all possible factors of 111111. 3

Ans : $111111 = 11 \times 10101$

$= 11 \times 3 \times 3367$

$= 11 \times 3 \times 7 \times 481$

$= 11 \times 3 \times 7 \times 13 \times 37$

$= 3 \times 7 \times 11 \times 13 \times 37$

possible factors of 111111 are 1, 3, 7, 11, 13, 37, 3×7, 3×11, 3×13, 3×37, 7×11, 7×13, 7×37, 11×13, 11×37, 13×37, 3×7×11, 3×7×11, 3×7×13, 3×7×37, 3×11×13, 3×11×37, 3×13×37, 7×11×37, 7×13×37, 11×13×37, 3×7×11×13, 3×7×11×37, 3×11×13×37, 3×7×13×37, 7×11×13×37, 111111.

i.e. 1, 3, 7, 11, 13, 37, 21, 33, 39, 111, 77, 91, 259, 143, 407, 481, 231, 273, 777, 507, 429, 1221, 1001, 2849, 3367, 5291, 3003, 8547, 10101, 15873, 37037, 111111.

The sum of the factors is 203352.

(b) In a book containing 20 pages, the average of the page numbers is 10. Later, it was detected that a sheet was missing from the book. Find the page numbers written on both sides of the missing sheet. 4

Ans : The question has a mistake. The average of the page numbers should be 11 in stead of 10.

Then total sum of page numbers in the book is $20\times11 = 220$.

But it was later delected that a sheet was missing.

Therefore the book actually contained 22 page numbers from 1 through 22.

Hence actual sum of the page numbers is

$$1+2+3+ \ldots\ldots\ldots + 22 = \frac{22\times23}{2} = 253$$

Hence the sum of the page numbers in the missing sheet is
253–220= 33

Hence page numbers on the missing sheet are 16 and 17.

(c) Examine if the product of two digit numbers AB and CD can be equal to a four digit number EEFF or not. 3

Ans : Clearly EEFF is divisible by 11. But neither AB nor CD is divisible by 11 since any two digit number divisible by 11 is of the from GG.

Hence EEFF cannot be product of two digit numbers of the form AB and CD.

5. (a) A, B, C, D are four consecutive digits of which A is the least. Determine the digit values of A, B, C, D, E, F from the following sum– 3

A B C D
A B C D
A B C D
A B C D
D E C F

Ans : Since A, B, C, D are consecutive digits with A as the least, therefore, we can take numbers in one of the following orders–

1, 2, 3, 4; 2, 3, 4, 5; 3, 4, 5, 6; 4, 5, 6, 7; 5, 6, 7, 8 and 6, 7, 8, 9.

Now,
1234
1234
1234
1234
4934

gives the only possible combination satistying the given sum.

Thus A = 1, B = 2, C = 3, D = 4, E = 9, F = 6.

(b) Three boys A, B, C were asked to divide a certain number by 1001 by the method of factors. They took the factors in the ratio 13, 11, 7; 7, 11, 13 and 11, 7, 13 respectively. If the first boy A obtained 3, 2, 1 as successive remainders find the successive remainders obtained by other two boys B and C. 7

Ans : We have $1001 = 13\times11\times7$

When the given number is divided by 13, 11, 7 by A, the remainders are 3, 2, 1.

Hence the number is

$13a+3$ with $a = 11b+2$ and $b = 7c+1$

$= 13\,(11b+2)+3$

$= 13\times11b+26+3$

$= 13\times11\,(7c+1)+29$

$= 13\times11\times7c+143+29$

$= 13\times11\times7c+172$

Next, B considers the factors in the order 7, 11, 13.

We have

$172 = 7\times24+4$

$24 = 11\times2+2$

$2 = 13\times0+2$

Hence the successive remainders when the given number is divided by 7, 11, 13 are 4, 2, 2.

Finally, $172 = 11\times15+7$

$15 = 7\times2+1$

and $2 = 13\times0+2$

Hence the successive remainders are 7, 1, 2.

6. (a) A worker was engaged for a certain number of days and was promised to be paid ` 1189. He remained absent for some days and was paid Rs. 1073 only. What was his maximum daily wage ? 5

Ans : Let the worker be engaged for n days.

Also, we suppose that the worker remained absent for x days.

Then his daily wage was

$$\frac{1189}{n}$$

and $$(n-x)\frac{1189}{n} = 1073$$

i.e. $$1189 - 1073 = \frac{x}{n}1189$$

i.e. $$\frac{x}{n} = \frac{116}{1189}$$

i.e. $$\frac{x}{116} = \frac{n}{1189}$$

i.e. $$\frac{x}{4} = \frac{n}{41}$$

The worker was engaged for a minimum of 41 days at a maximum of daily wage $\frac{1189}{41} = 29$ (rupees).

(b) Find the digit in the unit place of 777^{777} 5

Ans : 777^{777}

We have $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$.

After every fourth power of 7 we get 1 in the unit's place.

Now, $777 = 4\times194$

Thus,

$$(777)^{777} = (777)^{4\times194}$$

$$= \left\{(777)^4\right\}^{194}$$

Clearly unit's digit in 777^4 is 1 and therfore unit's digit in $\left\{(777)^4\right\}^{194}$ is also 1. i.e. $(777)^{777}$ has 1 in its unit's place.

7. (a) Three equal glasses are filled with mixture of milk and water. The ratio of milk to water in the first glass is as 2:3, in the second glass as 3:4 and in the third glass as 4:5. The contents of three glasses are emplied into a single bucket. What is the ratio of milk and water in the bucket? 5

Ans : Let x litre be the volume of the content of milk and water in each glass.

Then milk and water in the first glass are $\frac{2}{5}x$ and $\frac{3}{5}x$ respestively

similarly milk and water in 2nd and 3rd glasses are $\frac{3x}{7}$, $\frac{4x}{7}$ and

$\frac{4x}{9}$, $\frac{5x}{9}$ respectively.

Thus, amount of milk in the bucket is

$$\frac{2}{5x}+\frac{3x}{7}+\frac{4x}{9}=\frac{126x+135x+140x}{5\times7\times9}$$

$$= \frac{421}{5\times7\times9}x$$

And, the amount of water in the bucket is

$$\frac{3x}{5}+\frac{4x}{7}+\frac{5x}{9}=\frac{189x+180x+175x}{5\times7\times9}$$

$$= \frac{544x}{5\times7\times9}$$

Hence, the natio of milk and water is the bucket is

$$\frac{421}{5\times7\times9}x : \frac{544}{5\times7\times9}x$$

$= 421 : 544$

(b) 5 men and 2 boys together in 1 hour do 4 times of the work as 1 man and 1 boy can do in 1 hour. Determine the ratio of the specified times of works of the man and the boy. 5

Ans : Let as assume that a given work is done by a man in x hours while the same is done by a boy in y hours.

Hence in 1 hour a man and a boy do $\frac{1}{x}$ and $\frac{1}{y}$ parts of the work respectively.

So, together the man and the boy do $\frac{1}{x}+\frac{1}{y}$ part of the work in 1 hour.

Also, 5 men and 2 boys will do $\frac{5}{x}+\frac{2}{y}$ part of the work in 1 hour.

By condition

$$\frac{5}{x}+\frac{2}{y}=4\left(\frac{1}{x}+\frac{1}{y}\right)$$

$$\frac{5}{x}-\frac{4}{x}=\frac{4}{y}-\frac{2}{y}$$

i.e $\frac{1}{x}=\frac{2}{y}$

i.e $2x = y$

i.e $\frac{x}{y}=\frac{1}{2}$

Hence the boy will take double the time taken by the man to complete the given work.

8. (a) A man is standing on a railway bridge which in 50 meters long. He finds that a train crosses the bridge in 4 seconds but himself in 2 seconds. Find the length of the train and its speed. 5

Ans. : If the train moves x meters per second, then the length of the train will be 2x meters.

By given condition,

$4x=x+50$

i.e. $3x=50$

or $x=\frac{50}{3}$ meters.

i.e. length of the train is $\frac{50}{3}$ metres and velocity is–

$$\frac{\frac{50}{3}\times 60\times 60}{1000}=60 \text{ km per hour}$$

(b) A number of mangoes is to be divided amongst 3 persons so that one may get $\frac{5}{12}$ of it, another $\frac{7}{18}$ and the third the remainder, what must the number atleast be that this may be done without cutting any of the mangoes. 5

Ans. : First gets $\frac{5}{12}$ and second gets $\frac{7}{18}$ of the whole quantity.

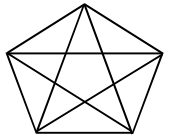
Therefore, the third gets $1-\frac{5}{12}-\frac{7}{18}$ of the whole quantity.

i.e. The parts of the three people are–

$\frac{5}{12}$, $\frac{7}{18}$ and $\frac{36-15-14}{36}=\frac{7}{36}$

Clearly, the least number of mangoes must be 36.

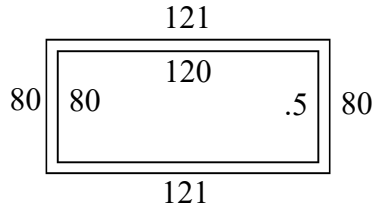
9. (a) Count all triangles in the following figure– 5



Ans. : The total number of triangle in the figure is 35.

(b) A rectangular garden is surrounded by a wall 5 meters high and 5 decimeters thick. How many bricks each 3.35 decimeters by 2 decimeters by 1 decimeter will be required to build the wall if the measurements of the garden are 120 meter by 80 meters inside the wall. 5

Ans. :



Volume of the wall is $(121\times80)\times2\times5$ m^3

Volume of a brick is $.335\times.2\times.1$ m^3

$$\text{Hence number of bricks} = \frac{2\times201\times5}{.335\times.2\times.1}$$

$$= \frac{201000000}{670}$$

$$= 300000$$

10. Making observations on the number of diagonds of regular polygons in simple cases like square, regular pentagon, regular hexagon etc. or otherwise estimate the number of diagonals of a regular decagon (diagonal of a polygon is a line segment joining any two of its non adjacent points. 10

Ans. : No. of diagonals in a square = 2
No. of diagonals in a pentagon = 5
No. of diagonals in a hexagon = 9
No. of diagonals in a heptagon = 14
No. of diagonals in a octagon = 20
No. of diagonals in a nonagon = 27
No. of diagonals in a decagon = 35